GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION
SPONSORED PROJECT INITIATION

853-362

Date: 1/18/79

Project Title: Optimal Coordination of Energy Storage Facilities with an Integrated Electric Power System

Project No: E-21-651 Green card

Project Director: Dr. A. P. Meliopoulos

Sponsor: National Science Foundation

Agreement Period: From 12/1/78 Until 2/28/81 (Grant Period)

Type Agreement: Grant No. ENG-7816496

Amount: $46,293 NSF
4,520 GIT (E-21-339)
$50,813 Total

Reports Required: Annual Progress Report(s); Final Project Report

Sponsor Contact Person (s):

Technical Matters

(Program Official)
Dr. William L. Brogan
System Theory and Application Program
Electrical Sciences and Analysis Section
Division of Engineering
National Science Foundation
Washington, D. C. 20550
202/632-5881

Contractual Matters
(thru OCA)

(Grants Official)
Ms. Mary Frances O'Connell
Grants Specialist, Area 4
MPE/BBS/SE Branch
Division of Grants and Contracts
National Science Foundation
Washington, D. C. 20550
202/632-2858

Defense Priority Rating: n/a

Assigned to: Electrical Engineering (School/Laboratory)

COPIES TO:

- Project Director
- Division Chief (EES)
- School/Laboratory Director
- Dean/Director—EES
- Accounting Office
- Procurement Office
- Security Coordinator (OCA)
- Reports Coordinator (OCA)
- Library, Technical Reports Section
- EES Information Office
- EES Reports & Procedures
- Project File (OCA)
- Project Code (GTRI)
- Other

CA-3 (3/76)

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION

Date: April 6, 1981

Project Title: Optimal Coordination of Energy Storage Facilities with an Integrated Electric Power System

Project No: E-21-651

Project Director: Dr. A. P. Meliopoulos

Sponsor: National Science Foundation

Effective Termination Date: 2/28/81

Clearance of Accounting Charges: 2/28/81

Grant/Contract Closeout Actions Remaining:

_ Final Invoice and Closing Documents
X Final Fiscal ~~Report~~ Accounting (FCTR)
X Final Report of Inventions
_ Govt. Property Inventory & Related Certificate
_ Classified Material Certificate
_ Other ________

Assigned to: Electrical Engineering (School/Laboratory)

COPIES TO:

Administrative Coordinator
Research Property Management
Accounting Office
Procurement Office
Research Security Services
Reports Coordinator (OCA)

Legal Services (OCA)
Library, Technical Reports
EES Research Public Relations (2)
Project File (OCA)
Other: ________

Progress Report

National Science Foundation Grant No. ENG-7816496

OPTIMAL COORDINATION OF ENERGY STORAGE FACILITIES WITH AN INTEGRATED ELECTRIC POWER SYSTEM

Introduction

The research program has as its overall objective the development of efficient models for the coordination of energy storage devices with an integrated hydrothermal electric power system. Such models are necessary for both expansion planning of electric power systems and operations planning. The ever increasing operating costs of electric power systems make energy storage devices economically attractive. One type of energy storage devices, namely pumped hydrostorage plants, have proven to reduce overall cost of electric power systems. In the future other types may also become economical. The results of this research program will provide efficient methodologies for projecting the impact of such devices on overall system costs. In addition the results of this research program will provide efficient methodologies for the optimal on-line control of energy storage devices by defining optimal control policies.

This section is a brief progress report defined in general terms. Detailed description of the work done is provided in four accompanied Appendices. This organization of the progress report seems to be practical because of the complexity of the models involved. Thus in the next two sections the Work Performed and

Work Remaining to Be Done are defined in general terms with references to the Appendices.

Work Performed

A formulation of the problem which leads to efficient computational procedure has been completed. Specifically the problem of coordinating an energy storage facility with an electric power system has been formulated as an optimal stochastic control problem. The objective of this problem is the minimization of the expected value of the restitution cost. The restitution cost is defined as the cost of energy stored during the charging cycle of the facility minus the energy replacement cost during the discharging cycle. The restitution cost depends on system electric load, available generating units and generating unit operating cost. Electric load and generating units are modeled as stochastic processes. The formulation of the problem as an optimal stochastic control problem and the models of electric load and generation system are given in Appendices A, B, and C respectively. Dynamic programming techniques have been applied to the optimal stochastic control problem defined in Appendix A. These techniques led to recurrence formulae which are of the quadratic programming variety.

The optimal stochastic control problem formulated in Appendix A is quite difficult to solve. Alternative solution procedures and approximations are being investigated. Specifically the following approaches are being examined:

1) Pure Feedback Approach

2) Pure Certainty Equivalent Approach

3) Certainty Equivalent Feedback Approach

4) Pure Open Loop Approach

5) Open Loop Feedback Approach

In particular in the Pure Certainty Equivalent Approach all stochastic processes at any time t are replaced with their corresponding expected values at time t. Thus a deterministic optimal control problem results. This approach has been examined first because of its simplicity. Efficient solution techniques have been developed for this case. They are presented in Appendix D. As the project continues the other approaches will be examined.

Finally, several computer programs have been developed for the purpose of studying the behavior of the models. In particular the following four computer programs have been developed:

a) NSFA. This program computes the incremental production cost of an integrated hydroelectric power system. A Markov model is employed for the generating units. The model is described in Appendix C.

b) NSFB. This program identifies an appropriate stochastic model for the electric load from given historical data. The methodology is described in Appendix B.

c) NSFC. This program solves a general quadratic programming problem. A simplex like algorithm is employed. Bounds on the variables are implicitly accounted for. In addition the program employs sparcity techniques. These features render the program a high level of efficiency. Appendix D describes the developed algorithm.

d) NSFD. This program solves the deterministic optimal control problem; that is the problem in which all the stochastic processes at time t are replaced with their expected values at time t (Certainty Equivalent Approach).

In summary models and associated computer programs have been developed for the optimal coordination of energy storage facilities with an integrated electric power system. By the time of this writing the certainty equivalent approach has been implemented and studied.

An initial report of this work has been submitted and accepted for presentation at the 18th IEEE Conference on Decision and Control, held in Fort Lauderdale, Florida, December 12-14, 1979. A copy of the presented paper is attached.

Work Remaining to Be Done

To this time much of the necessary modeling and computer program development has been done. Several other tasks remain to be done for the completion of this research program.

The first task is to validate the developed computer programs. It has become evident that efficiency of the programs is of great importance. Thus time will be devoted to increase the efficiency of the computer programs. It should be pointed out that this task is closely coupled to analytical developments.

The second task is to investigate alternative policies for the optimal coordination of energy storage facilities with an electric power system using the developed models. This task amounts to examining the following approaches for the solution of the optimal stochastic control problem formulated in Appendix A:

a) Pure Feedback Approach

b) Certainty Equivalent Feedback Approach

c) Pure Open Loop Approach

d) Open Loop Feedback Approach

The third task is to develop computer programs to investigate the performance of the mentioned approaches. These computer programs will directly utilize the computer program developments of the first year.

The fourth task is to investigate sensitivities of optimal policies to various system parameters and/or changes of operating conditions.

The fifth task is to publish the results of this research in refereed journals. The journal deemed most appropriate is the IEEE Transactions on Automatic Control.

Appendix A

This Appendix describes the formulation of the problem of optimal coordination of energy storage facilities with an electric power system and a decomposition technique of the problem into a series of smaller dimension problems. The formulation results in an optimal stochastic control problem. Application of the decomposition technique (dynamic programming) results in a recurrence formula of the quadratic programming variety. Subsequent sections describe the formulation of the problem and the decomposition technique.

Formulation

The problem of optimal coordination of energy storage devices, over a time period T, with the integrated electric power system aims primarily at the minimization of the overall system production cost. Constraints of energy availability, equipment capacities and electric power demand are imposed. The point of departure in formulating the problem is the chronological system load curve which is discretized as in Figure A.1. The time period is divided into N intervals of small duration (less than an hour). It is then assumed that the total system load remains constant for the duration of an interval. If x_i and y_i are the decision variables in interval i, defined as:

x_i charge level (mw) of energy storage device, interval i

y_i discharge level (mw) of energy storage device, interval i

then the impact of the control variables in the overall production cost of the system can be approximated with a quadratic function

$$f_i(x_i,y_i,L_i,S_{Ai}) = c_i^c x_i + d_i^c x_i^2 - c_i^d y_i + d_i^d y_i^2 \qquad (A.1)$$

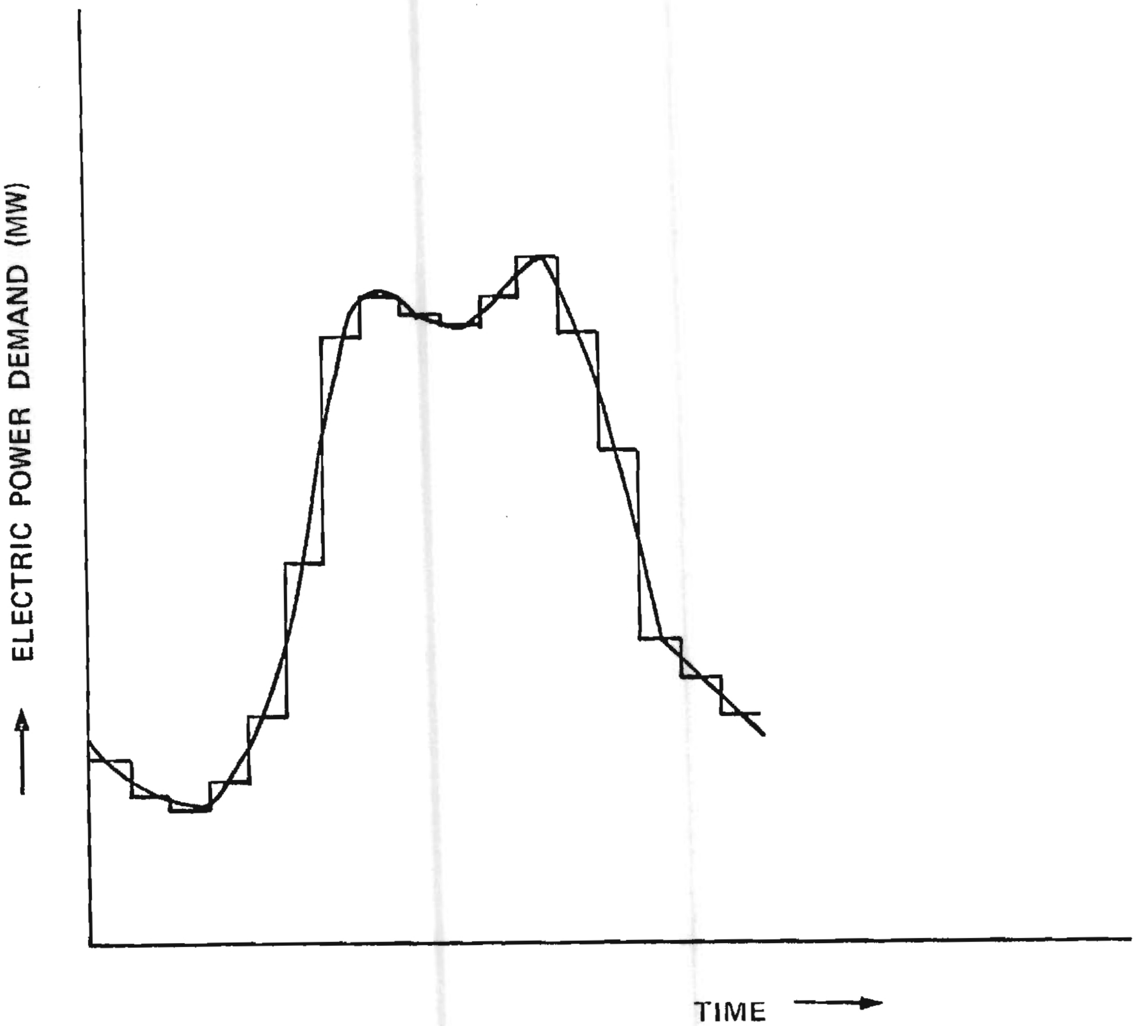


Figure A.1 Discretization of the Chronological Electric Load Curve

where

$$c_i^c, c_i^d, d_i^c, d_i^d \geq 0$$

are constants depending on total system load L_i and characteristics of available generating units, S_{Ai}. In addition certain constraints apply. Let S_i^c be the set of constraints for interval i. Then

$$S_i^c = \begin{cases} E_i = E_{i-1} + a_i x_i - b_i y_i + e_i - \ell_i \\ E_{min} \leq E_i \leq E_{max} \\ \underline{x}_i \leq x_i \leq \overline{x}_i \text{ or } x_i = 0 \\ \underline{y}_i \leq y_i \leq \overline{y}_i \text{ or } y_i = 0 \end{cases} \tag{A.2}$$

where

E_{min}, E_{max}	:	Minimum and maximum capacity of the energy storage device
$\underline{x}_i, \overline{x}_i, \underline{y}_i, \overline{y}_i$	:	operating limits on charging and discharging equipment
a_i, b_i	:	constants related to the efficiency of the device
e_i	:	externally available energy (such as run of the river for a hydro plant)
ℓ_i	:	losses in interval i
E_{i-1}	:	energy stored in device at the end of interval i-1.

At this point it should be pointed out that uncertainty is associated with the load L_i of the system and the set of available units S_{Ai}. Uncertainty is also associated with the availability of the energy storage device itself. The uncertainty is modeled as follows: An autoregressive model is developed and identified for the system load. This model is described in Appendix B. The uncertainty associated with the set S_{Ai} is modeled as follows: A Markov model is assumed for the individual generating units of the system. Then the statistics of the set S_{Ai} can be developed. This model is described in Appendix C. In the same Appendix the model for the computation of the parameters $c_i^c, c_i^d, d_i^c, d_i^d$ is also described. These parameters depend on the stochastic processes L_i and S_{Ai}.

With above definitions the problem of optimal stochastic coordination of energy storage devices with the electric power system can be stated as follows:

$$\text{Minimize } J = E \sum_{i=1}^{N} f_i(x_i, y_i, L_i, S_{Ai}) \tag{A.3}$$

$$\text{subject to } E_N = C \tag{A.4}$$

$$\text{and constraint set } S_i^c,\ i = 1,2,\ldots,N \tag{A.5}$$

where S_i^c is defined with (A.2)

The quantity $\sum_{i=1}^{N} f_i(x_i, y_i, L_i, S_{Ai})$ is called the restitution cost because it equals the cost of energy stored during the charging cycles minus the energy replacement cost during the discharging cycles. The expectation E is taken with respect to the stochastic processes L_i, S_{Ai}. In above formulation the terminal condition $E_N = c$ is assumed. This is not restrictive. An initial condition $E_o = c_o$ may be assumed or none, as well. The terminal condition is assumed for practical reasons.

The formulated problem is rather general. It applies to any energy storage device which can be coordinated with the electric power system with single or multiple reservoirs. It also applies to the coordination of conventional hydroelectric plants with the power system. Because of the large dimensionality of the model and its stochastic nature, a direct solution is not feasible. The model however possesses certain properties which enable the decomposition of the problem into smaller problems, easily solvable. The decomposition leads naturally to a dynamic program. The associated recurrence equation can be solved via quadratic programming. In the subsequent section, the decomposition technique is discussed.

Decomposition

The model possesses the following properties:

* the objective function is separable
* the constraints are also separable

For this model the following decomposition scheme is natural: Consider the time period T which has been divided into N intervals. Assume the period T is divided into K subperiods. A subperiod is defined with the set I_k of successive intervals:

$$I_k=\{i;\ i = n_{k-1} + 1,\dots,n_k\} \tag{A.6}$$

$$k = 1,2,\dots,K$$

$$n_o = 1$$

$$\eta_K = N$$

Next define the following subproblem:

$$\text{Minimize } J_\lambda = E\left\{\sum_{i=\eta_{\lambda-1}+1}^{N} f_i(x_i, y_i, L_i, S_{Ai})\right\} \tag{A.7}$$

$$\text{subject to } E_N = c = \xi_K \tag{A.8}$$

$$\text{and constraint set } S_i^c,\ i = \eta_{\lambda-1}+1, \ldots, N \tag{A.9}$$

$$\text{and parameter } \xi_\lambda = E_{\eta_{\lambda-1}}$$

Let the optimal solution to this problem be

$$\Lambda^*_\lambda(\xi_\lambda, \xi_K)$$

Clearly the optimal solution is a function of the terminal conditions ξ_λ, ξ_K.

Applying dynamic programming techniques the following recurrence formula can be developed:

$$\Lambda^*_\lambda(\xi_\lambda, \xi_K) = \min_{\xi_{\lambda+1}} E\left\{\Lambda^*_{\lambda+1}(\xi_{\lambda+1}, \xi_K) + \sum_{i\varepsilon I_\lambda} f_i(x_i, y_i, L_i, S_{Ai})\right\} \tag{A.10}$$

subject to

$$\text{constraint set } S_i^c,\ i\varepsilon I_\lambda \tag{A.11}$$

$$\text{and parameter } E_{\eta_{\lambda-1}} = \xi_\lambda \tag{A.12}$$

The expectation in the above problem is taken with respect to L_i and S_{Ai}.

The solution to the above problem is obtained as follows: For a fixed value of the variable $\xi_{\lambda+1}$ the problem defined with the recurrence formula (A.10), and constraints (A.11) and (A.12) collapses to an optimization problem of the quadratic programming variety. This problem is further examined in Appendix D. The conclusion here is that the right hand side of the recurrence formula (A.10) is a computable functional of the parameter $\xi_{\lambda+1}$. The optimal value $\xi^*_{\lambda+1}$ can be found with a direct search or using sensitivity analysis. In any case a large number of quadratic programming problems need to be solved. For this reason emphasis was placed on the efficiency of the quadratic program. A highly efficient procedure has been developed which has been implemented with sparcity techniques. This procedure is described in Appendix D.

In conclusion the problem of coordination of energy storage devices has been formulated as an optimal stochastic control problem. Solution of the optimal stochastic control problem is obtained with dynamic programming techniques. These techniques lead to recurrence formulae which are of the quadratic programming variety. Solution techniques for the recurrence formulae are examined in Appendix D.

APPENDIX B

Electric Load Model

This Appendix provides a description of the stochastic model employed for the electric load. The described model has been selected after a thorough investigation and evaluation of existing models. The model is suitable for the purposes of this research project.

Analysis of historical data of electric load, $\ell(t)$, indicates that the electric load is a nonstationary stochastic process. Modelling and identification of nonstationary stochastic processes is quite difficult. Fortunately, it has been observed that an appropriate differential of the nonstationary stochastic process $\ell(t)$ may behave as a weakly stationary stochastic process. For example

$$x(t) = \ell(t) - \ell(t-2).$$

Because the electric load, $\ell(t)$, exhibits daily, weekly, and seasonal periodicity it is expedient to define a transformation of $\ell(t)$ into an assumed weakly stationary stochastic process $x(t)$, as follows:

$$x(t) = \nabla^d \nabla_s^D \ell(t)$$

where

$\nabla = 1-B$	is a nonseasonal backward difference operator
$\nabla_s = 1-B^s$	is a seasonal backward difference operator
d	is the degree of nonseasonal differencing
B	is a backward operator defined with $B\ell(t) \triangleq \ell(t-1)$
D	is the degree of seasonal differencing

A stationary stochastic process, such as $x(t)$, can be generated if white noise is supposed to be transformed by a linear filter as in Figure B.1.

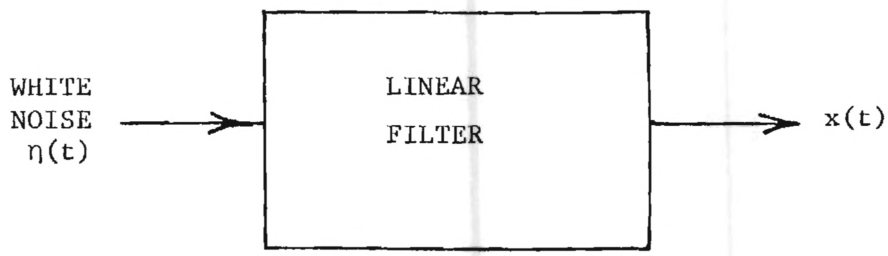


Figure B.1. Generation of a Stationary Stochastic Process

The white noise is defined as follows:

$$E[\eta(t)] = 0$$

$$\mathrm{Cov}[\eta(t)\ \eta(t+\tau)] = \begin{cases} \sigma_\eta^2 & \text{if } \tau = 0 \\ 0 & \text{otherwise} \end{cases}$$

For the present application the linear filter is so selected as to result to an autoregressive model for the stochastic process $x(t)$:

$$\sum_{i=0}^{n} a_i\ x(t) + \eta(t) = 0 \quad , \quad a_o = 1$$

The defined model involves n parameters a_i, $i = 1, \ldots, n$, which are estimated via least square estimation. Once the model has been identified, it is evaluated to determine level of uncertainty. This is done by computing the standard deviation of the prediction.

If the level of uncertainty is acceptable, the model is complete. Otherwise another model has to be selected and the procedure is repeated. Table B.1 summarizes the procedure.

The described procedure in Table B.1 has been implemented. Preliminary results indicate that a low order model provides good fit to historical data. The historical data used were obtained from the local utility.

Table B.1. Development of a Load Model

Step 1. Collect Load Historical Data $\ell(t)$, $t = -m, -m+1, \ldots, 0$

Step 2. Select the Order of the Model

$$d,\ D,\ s, \text{ and } n$$

Step 3. Estimate the Parameters $a_1, a_2, \ldots, a_n$ via Least Square Estimation

Step 4. Compute level of uncertainty (Standard Deviation of Prediction)

If Level of Uncertainty is Acceptable, stop. Otherwise select new Model Parameters (d, D, s, and n) and Go to 3.

Appendix C

Incremental Production Cost Stochastic Model

This Appendix describes the development of a stochastic model for the computation of incremental production cost of an electric power system. The incremental production cost is essential in the optimal coordination of energy storage facilities with an electric power system.

It is shown that optimal load flows provide the incremental production cost at the busbars of energy storage facilities (lagrangian multipliers). Optimal load flows, however, are computationally unacceptable for the pruposes of this research project. A good approximation is provided with the so-called busbar economic dispatch. The busbar economic dispatch provides the incremental production cost directly as a function of system load and generating unit operating costs. Because generating units are subject to forced outages a stochastic model is assumed for the generation system.

Detailed description of models follow:

Incremental Production Cost

Central to the computation of optimal policies for the operation of energy storage devices is the knowledge of electric energy production cost of the integrated electric power system. In particular it is necessary to compute the incremental cost of electric energy at the busbar of energy storage devices. This section examines the problem of computing the incremental cost of electric energy at the busbar of energy storage devices.

Consider a general electric power system as in Figure C.1. It consists of M generating units and N loads, S energy storage devices, and the interconnecting transmission system. Our objective is to determine the incremental cost of energy at busbars 1, 2, ..., s. To this purpose we start with the formulation of the operating constraints and objectives of an electric power system. The result is the so called optimal load flow which is written here in the following form

$$\text{Minimize } C_o = \sum_{i=1}^{M} f_i(u_i) \tag{C.1}$$

$$\text{subject to} \quad g(x, u, v) = 0 \tag{C.2}$$

$$h(x, u, v) \leq 0 \tag{C.3}$$

where

$f_i(u_i)$ is the operating cost of plant i, output u_i

x is the state representation of the electric power system

v is the vector of the level operation of the energy storage devices

$g(x,u,v) = 0$ load flow equations

$h(x,u,v) \leq 0$ operating constraints on system components.

The solution to this problem is secured with the Kuhn-Tucker conditions:

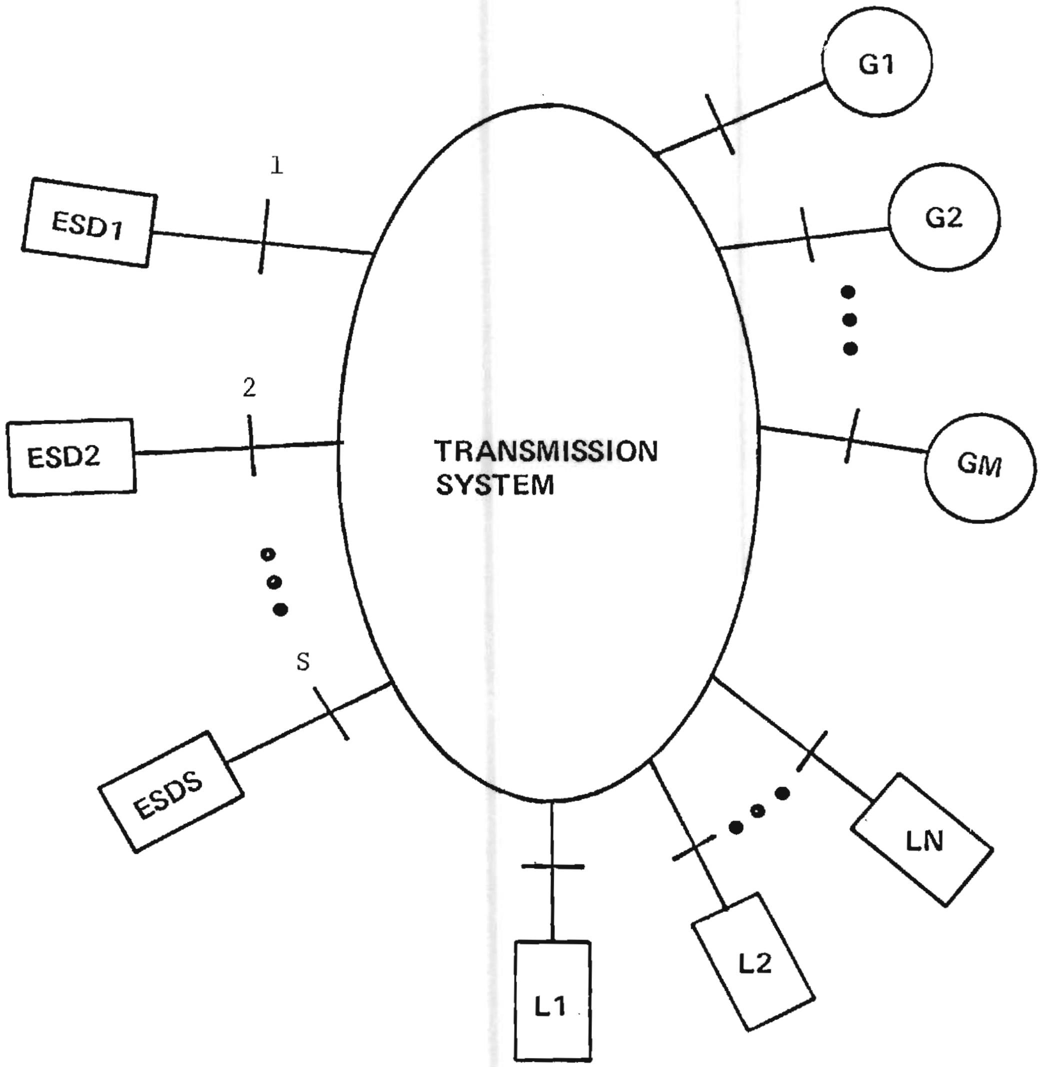


Figure C.1. An Integrated Electric Power System.

$$\mu \geq 0$$

$$\nabla C_o + \lambda^T \nabla g + \mu^T \nabla h = 0$$

$$g(x,u,v) = 0$$

$$h(x,u,v) \leq 0$$

$$\mu^T h(x,u,v) = 0$$

where ∇ is the gradiant with respect to x and u.

If the load flow equations (C.2) are arranged such that the first equation expresses the real power balance at busbar 1, etc., and the Sth equation expresses the real power balance at busbar S, then the lagrangian multipliers $\lambda_1, \lambda_2, \ldots, \lambda_s$ provide the incremental cost of power at busbars 1, 2, ..., s. If $C_i(x,u,v)$ is the cost of operation of energy storage device i at level v_i, then

$$\lambda_i \triangleq \frac{\partial c_i(x,u,v)}{\partial v_i}$$

This procedure is novel. Computer programs have been developed and used routinely by the industry for operational practices. However it is impractical for planning purposes because of excessive computations and large amounts of data required. A simplification to the problem results upon the observation that actual power systems have the following features:

1) Low losses (1 to 3 percent)

2) Strong transmission systems

Feature 2) suggests that constraints (C.3) are normally inactive, and feature 1) suggests that a good approximation will be to replace the load flow equations (C.2) with a simple equation:

$$\sum_{i=1}^{M} u_i + \sum_{i=1}^{S} v_i - L - P_L = 0 \tag{C.4}$$

$$\underline{u}_i \leq u_i \leq \overline{u}_i \quad \text{or} \quad u_i = 0 \quad i = 1, 2, \ldots, M \tag{C.5}$$

where

L is the total electric power load in the system

P_L is an approximate estimate of system losses (an educated guess), and it is considered constant

u_i is the output of plant i

v_i is the output of energy storage device i

Then the problem becomes

$$\text{Minimize} \quad C_o = \sum_{i=1}^{M} f_i(u_i) \tag{C.1}$$

$$\text{subject to:} \sum_{i=1}^{M} u_i + \sum_{i=1}^{S} v_i - L - P_L = 0 \tag{C.4}$$

$$\underline{u}_i \leq u_i \leq \overline{u}_i \quad \text{or} \quad u_i = 0 \quad i = 1, 2, \ldots, M \tag{C.5}$$

This problem is known as the busbar economic dispatch problem. The solution of above problem is obviously a function of $L' = L - \sum_{i=1}^{s} v_i$:

$$C_o = F(L')$$

The incremental energy cost for energy storage plant i is

$$\lambda_i = \frac{\partial C_o}{\partial v_i} = \frac{\partial F(L')}{\partial v_i} = \frac{\partial F(L')}{\partial L'} \frac{\partial L'}{\partial v_i} = - \frac{\partial F(L')}{\partial L'}$$

Note that for this simplified problem

$$\lambda_i = \lambda_j \qquad i,j = 1, 2, \ldots, s$$

That is the incremental production cost is same for every energy storage device. This is a result of the assumption of constant losses in the transmission network. This assumption will not introduce an appreciable error.

In summary, given a set of generating units {1, 2, ..., M} with their cost functions $f_i(u_i)$, $i = 1. 2, \ldots M$ and allowable operating limits $\underline{u}_i$, $\overline{u}_i$, $i = 1, 2, \ldots, M$, and s energy storage devices operating at levels v_i, $i = 1, 2, \ldots, s$ and a total load L of the system, the total operating cost is determined from the solution of the following problem:

$$\text{Minimize} \quad C_o = \sum_{i=1}^{M} f_i(u_i) \tag{C.1}$$

$$\text{subject to:} \sum_{i=1}^{M} u_i - L' - P_L = 0 \tag{C.6}$$

$$\underline{u}_i \leq u_i \leq \overline{u}_i, \quad \text{or} \quad u_i = 0 \quad i = 1, 2, \ldots, M \tag{C.5}$$

$$L' = L - \sum_{i=1}^{S} v_i \tag{C.7}$$

and the incremental power cost from

$$\lambda = - \frac{\partial C_o^*(L')}{\partial L'} \tag{C.8}$$

where $C_o^*(L')$ is the optimal solution of the problem defined with (C.1), (C.5), (C.6), and (C.7).

Random Outages of Generating Units

Generating units are subject to random events leading to their outages or their inability to generate electric power. Thus the availability of a generating unit is a stochastic process. Since the incremental production cost, λ, is dependent on the available generating units λ is also a stochastic process. The model of this stochastic process will be derived from a probabilistic model for the generating system. To this purpose let's consider a single generating unit consisting of m components. For a specific component define the conditional failure rate $\beta(t)$ (useful in reliability studies) as follows:

> Given a component subject to failures then $\beta(t)dt$ equals the probability that the given component fails in the interval $(t, t+dt)$, assuming that it did not fail up to time t.

The probability $\beta(t)dt$ may be expressed as

$$\beta(t)dt = f(t/T \geq t)\ dt$$

where $T \geq t$ means that component x has not failed up to time t (T is a random variable equal to the time of failure).

$$\text{Now } f(t/T \geq t)\ dt = \frac{\Pr\ [\text{failure in } t,\ t+dt]}{\Pr\ [\text{failure in } T \geq t]}$$

Let f(t) be the probability density function of system failure

Apparently

$$\int_{-\infty}^{\infty} f(t)\ dt = 1$$

Define the probability cumulative function

$$F(t) = \int_{-\infty}^{t} f(\tau)\ d\tau$$

Then

$$f(t/T \geq t)\, dt = \frac{f(t)\, dt}{\int_t^{\infty} f(\tau)\, d\tau} = \frac{\frac{dF(t)}{dt}\, dt}{1 - F(t)} = \frac{dF(t)}{1 - F(t)}$$

or

$$\beta(t)\, dt = \frac{dF(t)}{1 - F(t)}$$

Integration of above relationship from time 0 to t yields

$$F(t) = 1 - \exp\left\{ - \int_0^t \beta(\tau)\, d\tau \right\}$$

Assuming $\beta(t)$ to be constant (independent of time) we obtain

$$F(t) = 1 - \exp(-\beta t) \tag{C.9}$$

and

$$f(t) = \beta \exp(-\beta t) \tag{C.10}$$

In summary a specific component of a generating unit is subject to failures. Equations (C.9) and (C.10) describe the statistics of failures. Looking at a generating unit as a whole we observe that if a given set of components is simultaneously operational (have not failed) then the unit will be operational with maximum generation capability P. In general n(i) such states can be identified with generating unit i. There is a corresponding probability $p_i(j)$ that unit i is in state j with maximum generating capability $P_i(j)$. Thus for a specific time t the probabilistic model of a generating unit is summarized as follows:

$$\Pr[A_i \leq x] = \int_{-\infty}^{x} \sum_{j=1}^{n(i)} p_i(j)\ \delta(\tau - P_i(j))\ d\tau$$

where A_i is a random variable describing the available generating capacity of unit i.

Because of failures of specific equipment and similarly because of repairs on specific failed equipment, the state of a generating unit may change in time. Define

$p_i(j,t)$ to be the probability that unit i is in state j at time t.

Then there is an associated probability that unit i will change state in a time period dt. This is illustrated in Figure C.2. Define

$\lambda_i(j,k)$ to be the conditional transition rate from state j to state k

Then

$\lambda_i(j,k)\ dt$ is the probability that the unit goes to state k in the interval (t, t + dt), assuming that it was in state j at time t.

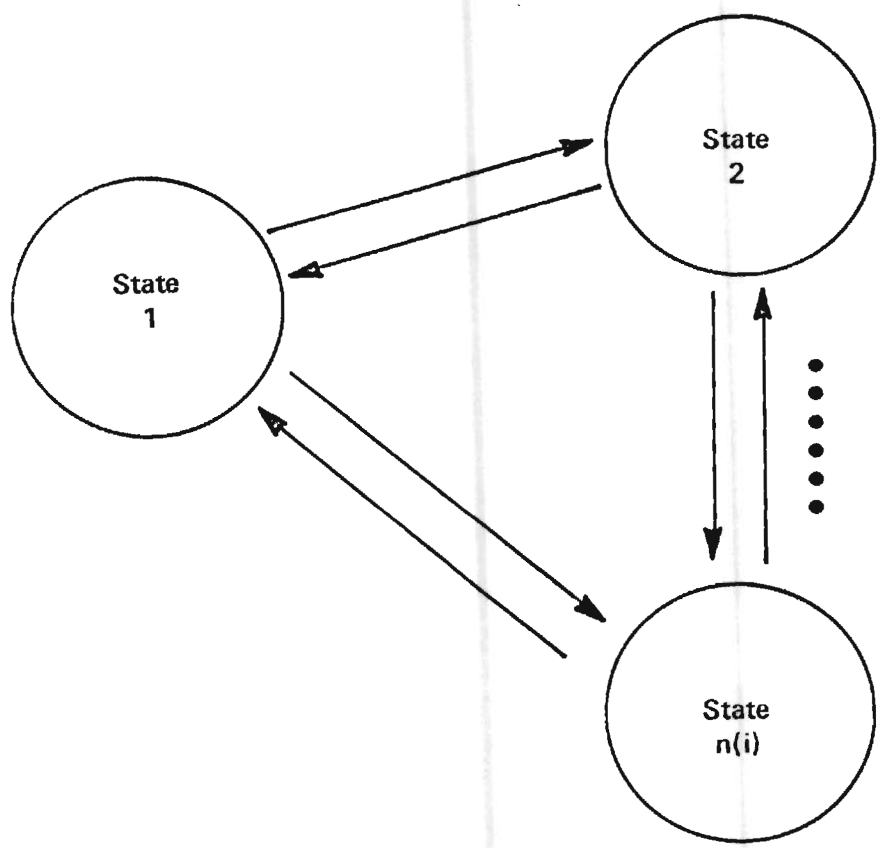


Figure C.2. Multistate Model of Availability of a Generating Unit.

With above definitions a Markor model can be generated for unit i as follows:

Assume that the probabilities

$$p_i(j,t) \qquad j = 1, 2, \ldots, n(i)$$

are known at time t. Compute the same probabilities at time t + dt. Obviously:

$$p_i(j,t+dt) = p_i(j,t)(1-\lambda_i(j,j)dt) + \sum_{\substack{k=1\\k\neq j}}^{n(i)} p_i(k,t)\lambda_i(k,j)dt$$

$$j = 1, 2, \ldots, n(i)$$

By rearranging and dividing by dt we obtain

$$\frac{dp_i(j,t)}{dt} = -p_i(j,t)\lambda_i(j,j) + \sum_{\substack{k=1\\k\neq j}}^{n(i)} p_i(k,t)\lambda_i(k,j) \qquad (C.11)$$

In addition

$$\sum_{j=1}^{n(i)} p_i(j,t) = 1.0 \qquad \text{for any } t.$$

Given an initial condition

$$p_i(j,0), \quad j = 1, 2, \ldots, n(i) \qquad (C.12)$$

one can integrate the equations (C.11) and (C.12) to obtain p(j,t) for any j and t.

In summary a general stochastic model has been developed describing the available generating capacity of a unit.

Generating System Stochastic Model

Given a generating system with m units and their associated stochastic models:

$$\{ n(i),\ p_i(j,t),\ j = 1, \ldots, n(i),\ \ P_i(j),\ j = 1, 2, \ldots, n(i) \}$$

$$i = 1, 2, \ldots, m$$

There are

$$N = \prod_{\text{all } i} n(i)$$

discrete states of the generating system. This is a large number of states. Most of these states, however, have a low probability of existence. Thus a tremendous reduction in the number of states is achieved if it is decided to disregard all states with probability less than a threshold value p_{th}. This value may be selected to be in the order of few hundredths.

The identification of the states with probability of existence greater than p_{th} is achieved as follows.

Consider state $X_{GK}(t)$ of the generating system. This state is defined as follows:

$$X_{GK} = \{(p_i(j(k),t),\ P_i(j(k),t)),\ \ i = 1, 2, \ldots, m\}$$

And the probability of existence is

$$Pr[X_{GK}] = \prod_{\text{all } i} p_i(j(k),\ t) \tag{C.13}$$

The state X_{GK} is rejected if

$$Pr[X_{GK}] < p_{th} \tag{C.14}$$

Implementation

The described model can be easily implemented. To this purpose consider a period of duration T. This period can be divided into K subperiods. Each subperiod may be several hours long. At this point the following reasonable assumption can be made: The state of the generation system may change only at the beginning of a subperiod. This assumption guarantees that the state of the generating system will not alter for the duration of one subperiod. Thus the incremental production cost during one subperiod, given the state of the generating system will be a function of system electric load only. This suggests the following computational procedure: Consider subperiod t. Using the Markov model the probabilities $p_i(j,t)$ can be computed via a numerical integration of the equations C.11) and (C.12). Then all the states of the generating system with probability greater than p_{th} can be computed using equations (C.13) and (C.14).

Finally for every state X_{GK} of the generating system with $P_r[X_{GK}] \geq p_{th}$, the incremental production cost versus system electric load can be computed using equations (C.8) after solving problem defined with (C.1), (C.5), (C.6) and (C.7). The procedure needs to be repeated for every subperiod t.

A computer program with the symbolic name NSFA has been developed for the computation of incremental production cost versus system electric load. The described Markov model of the generating system is employed. A flow chart of the program is given in Figure C.3.

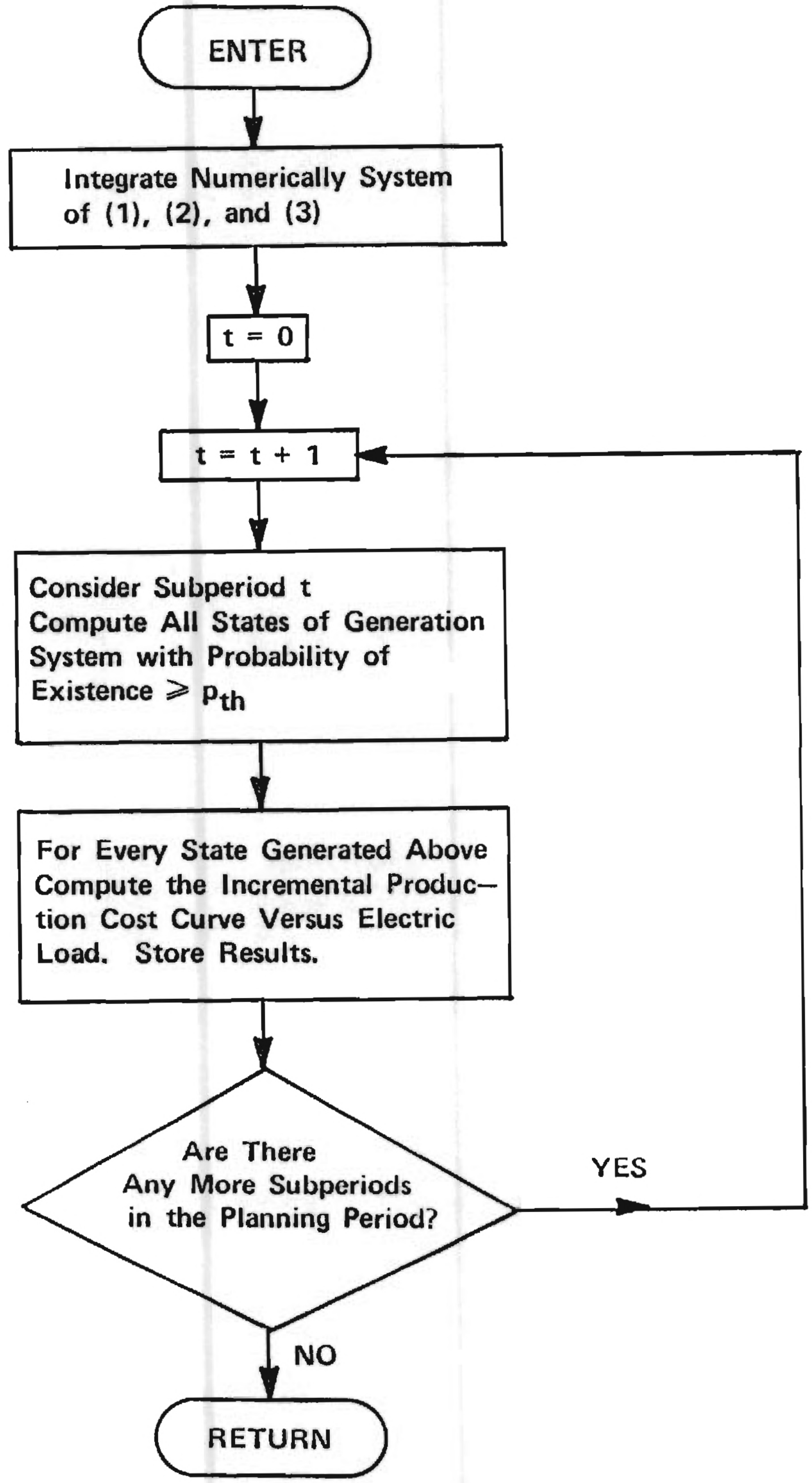


Figure C.3. Flow Chart of Computer Program NSFA.

Appendix D

This Appendix describes the solution of the problem defined with the recurrence formula (A.10) and constraints (A.11) and (A.12) of Appendix A. For a given set of parameters $\xi_{\lambda+1}$, L_i, and S_{Ai} the problem collapses to an optimization problem of the quadratic programming variety. In general the problem can be expressed as follows:

$$\text{Minimize } f(x) = C^T x + x^T Dx$$

$$\text{subject to} \quad A_i x \leq b_i$$

$$A_e x = b_e$$

$$x \leq h$$

$$x \geq 0$$

where x, c, b_i, b_e and j are vectors and A_i, A_e and D are matrices of appropriate dimensions.

In subsequent paragraphs the solution algorithm for this problem is described.

Quadratic Programming With Bounded Variables

Consider the following quadratic programming problem:

$$\text{Minimize} \quad C^T X + X^T DX \tag{D.1}$$

$$\text{subject to } A_i X \leq b_i \tag{D.2}$$

$$A_e X = b_e \tag{D.3}$$

$$X \leq h \tag{D.4}$$

$$X \geq 0 \tag{D.5}$$

where

X: $n \times 1$ charge/discharge vector

C: $n \times 1$ vector

D: $n \times n$ symmetric positive definite matrix

b_i: $m_i \times 1$ vector

b_e: $m_e \times 1$ vector

h: $n \times 1$ vector

A_i: $m_i \times n$ matrix

A_e: $m_e \times n$ matrix

This is equal to:

$$\text{Minimize:} \quad C^T X + X^T DX = f(x) \tag{D.6}$$

$$\text{subject to } g_1(x) = A_i X - b_i \leq 0 \tag{D.7}$$

$$h(x) = A_e X - b_e = 0 \tag{D.8}$$

$$g_2(x) = x - h \leq 0 \tag{D.9}$$

$$g_3(x) = -x \leq 0 \tag{D.10}$$

Applying Kuhn-Tucker conditions the problem is transformed into finding a feasible solution to the following set of equations and constraints:

$$\nabla f(x) + (\nabla h(x))^T \lambda + (\nabla g_1(x))^T \mu^1 + (\nabla g_2(x))^T \mu^2 + (\nabla g_3(x))^T V = o \tag{D.11}$$

$$C + 2DX + A_i^T\mu^1 + A_e^T\lambda + I\mu^2 - I\upsilon = 0 \tag{D.12}$$

$$A_iX + S^1 = b_i \tag{D.13}$$

$$A_eX = b_e \tag{D.14}$$

$$\mu^{2^T}(X - h) = \mu^{2^T}S^2 = 0 \tag{D.15}$$

$$\mu^{1^T}(A_iX - b_i) = \mu^{1^T}S^1 = 0 \tag{D.16}$$

$$\upsilon^TX = 0 \tag{D.17}$$

$$X,\ \upsilon,\ \mu^1,\ \mu^2,\ S^1,\ S^2 \geq 0 \tag{D.18}$$

$$\lambda \text{ free} \tag{D.19}$$

where

μ^1:	$m_i \times 1$ vector
μ^2:	$n \times 1$ vector
υ:	$n \times 1$ vector
λ:	$m_e \times 1$ vector
S^1:	$m_i \times 1$ vector
S^2:	$n \times 1$ vector

The following should be noted:

1. The free variables λ can be substituted with $\lambda = \lambda^+ - \lambda^-$ where $\lambda^+, \lambda^- \geq 0$ and λ^+, λ^- are $m_e \times 1$ vectors.
2. The equation $X + S^2 = h$ imposes upper bounds on the variables X. These constraints can be implicitly taken into consideration with the use of simple transformations as in the case of the simplex method with upper bounds for linear programming.

In matrix notation and with the above observations the problem is reduced to the following

$$\begin{bmatrix} -2D, & -A_i^T, & -A_e^T, & A_e^T, & -I, & I, & 0 \\ A_i, & 0, & 0, & 0, & 0, & 0, & I \\ A_e, & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ \mu^1 \\ \lambda^+ \\ \lambda^- \\ \mu^2 \\ \upsilon \\ S^1 \end{bmatrix} = \begin{bmatrix} C \\ b_i \\ b_e \end{bmatrix} \tag{D.20}$$

$$\upsilon^T X = 0 \tag{D.21}$$

$$\mu^{1T} S^1 = 0 \tag{D.22}$$

$$\mu^{2T} S^2 = \mu^{2T}(h-X) = 0 \tag{D.23}$$

$$X,\ \mu^1,\ \mu^2,\ \lambda^+,\ \lambda^-,\ \upsilon,\ S^1 \geq 0 \tag{D.24}$$

and upper bounds on variables X

$$X \leq h \tag{D.25}$$

The dimensions of the vector variables are:

x	μ^1	λ^+	λ^-	μ^2	υ	S^1
η	m_i	m_e	m_e	η	η	m_i

The above problem can be solved with the basic procedure of phase I of the simplex method as follows:

a) Introduce artificial variables to construct a starting feasible solution.

b) Set up a linear program with objective function equal to the sum of artificial variables and constraints (D.20), (D.24), and (D.25).

c) Utilize the simplex method with upper bounds to solve the problem defined in (b) with the following modification: A variable can not enter the solution basis unless it satisfies the exclusivity constraints (D.21), (D.22), and (D.23).

Development of the Algorithm

Let the following notation be introduced:

$$A = \begin{bmatrix} -2D, & -A_i^T, & -A_e^T, & A_r^T, & -I, & I, & 0 \\ A_i, & 0 & 0 & 0 & 0 & 0 & 0 \\ A_e, & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}.$$

$$y = \begin{bmatrix} X \\ \mu^1 \\ \lambda^+ \\ \lambda^- \\ \mu^2 \\ \upsilon \\ S^1 \end{bmatrix} \qquad b = \begin{bmatrix} C \\ b_i \\ b_e \end{bmatrix}$$

where

A : $(n + m_i + m_e) \times (3n + 2m_i + 2m_e)$ matrix

y : $(3n + 2m_i + 2m_e) \times 1$ vector

b : $(n + m_i + m_e) \times 1$ vector

The matrix A can be represented by column vectors a^i as follows

$$A = [a^1, a^2, \ldots, a^{3n + 2m_i + 2m_e}]$$

where all the vectors a^i are $(n + m_i + m_e) \times 1$.

If artificial variables r are introduced and the above notation used then the problem can be formulated as the following linear program.

$$\text{Minimize} \sum_{j=1}^{n+m_i+m_e} r_j = [c^1]^T \begin{bmatrix} Y \\ \\ r \end{bmatrix} \tag{D.26}$$

$$\text{subject to } AY + r = b \tag{D.27}$$

$$Y, r \geq 0 \tag{D.28}$$

exclusivity constraints

$$\upsilon_i X_i = Y_i Y_{i+\ell_1} = 0 \quad i = 1,2,\ldots,n \tag{D.29}$$

$$\ell_1 = 2n + m_i + 2m_e$$

$$\mu^1_{i-n} S^1_{i-n} = Y_i Y_{i+\ell_1} = 0 \quad i = n+1,\ldots,n+m_i \tag{D.30}$$

$$(h_i - X_i)\mu^2_i = (h_i - Y_i)Y_{i+\ell_2}$$

$$i = 1,2,\ldots,n \tag{D.31}$$

$$\ell_2 = n + m_i + 2m_e$$

$$X_i = Y_i \leq h_i \qquad i = 1,2,\ldots,n \tag{D.32}$$

where

C^1 : $(4n + 3m_i + 3m_e) \times 1$ vector

r : $(n + m_i + m_e) \times 1$ vector

The problem defined with objective (D.26) and constraints (D.27), (D.28), (D.29), (D.30), (D.31) and (D.32) can be solved as follows:

1) A starting extended basic feasible solution is given by

$$r = b$$
$$Y = 0$$

2) Select a variable, which if enters the solution will improve the objective function, and which will not violate the exclusivity constraints. Thus a variable k in order to qualify for entering variable has to pass the following two tests:

2a) Constraints (D.29), (D.30), and (D.31) are not violated.

2b) The reduced cost coefficient r_k is less than zero

$$r_k = (C_k^1 - C_B^{1T} B^{-1} a^k) e_k < 0$$

where

C_B^1 is the vector of the objective function corresponding to the basic variables. The i^{th} entry is one if the i^{th} basic variable is an artificial variable, otherwise zero

C^1_k is the k^{th} entry of the vector C^1.

B is a $(n + m_i + m_e) \times (n + m_i + m_e)$ matrix of the basis. The columns of B are the columns of the matrix A that corresponds to basic variables.

a^k is the vector of the matrix A corresponding to variable k.

$$e_k \begin{cases} +1 \text{ if variable } k \text{ is at the lower bound.} \\ -1 \text{ if variable } k \text{ is at the upper bound} \end{cases}$$

If no variable can be found that satisfies both tests then the algorithm terminates. If all the artificial variables r are zero then a solution has been found.

3) Find out which variable leaves the basis or if variable k goes to its opposite bound. Compute the following

$$\alpha = B^{-1} a^k$$

$$S = B^{-1} b$$

$$\varepsilon_1 = h_k$$

$$\varepsilon_2 = \min_i \left\{ \frac{S_i}{\alpha_i} \,,\quad \alpha_i > 0 \right\} \text{, index } j$$

$$\varepsilon_3 = \min_i \left\{ \frac{S_i - h_i}{\alpha_i} \,,\quad \alpha_i < 0 \right\} \text{ index } j$$

If $\varepsilon_1 \leq \varepsilon_2, \varepsilon_3$ go to 4

If $\varepsilon_2 < \varepsilon_1$ and $\varepsilon_2 \leq \varepsilon_3$ go to 5

If $\varepsilon_3 < \varepsilon_1, \varepsilon_2$ go to 6

4) The variable k goes to its opposite bound.

Change sign of variable e_k

$$e_k = - e_k$$

Update the b vector

$$b = b + h_k a^k e_k$$

Go to step 2

5) The j^{th} basic variable becomes nonbasic and returns to its old bound. The k^{th} variable enters the basis.

Replace the vector a^j in the B matrix with the vector a^k.

Go to step 2.

6) The j^{th} basic variable becomes nonbasic and goes to its opposite bound. The k^{th} variable enters the basis.

Change sign of variable e_j.

$$e_j = - e_j$$

Update vector b

$$b = b + h_j a^j e_j$$

Replace the vector a^j in the B matrix with the vector a^k.

Go to step 2.

The described algorithm has been implemented in a computer program. Because matrix B is highly sparce, sparcity techniques have been employed yielding a highly efficient program.

A DECOMPOSITION TECHNIQUE TO DETERMINE OPTIMAL COORDINATING POLICIES OF ENERGY STORAGE PLANTS WITH ELECTRIC POWER SYSTEM

A. P. Meliopoulos

Georgia Institute of Technology
School of Electrical Engineering
Atlanta, Georgia 30332

Abstract - The problem of coordinating energy storage plants with an integrated hydroelectric power system to achieve minimum overall operating costs is a large and complex stochastic decision problem. This paper describes a decomposition technique which reduces the complexity of the problem and results in practical computational algorithms. The method is based on the definition of two subproblems. The first subproblem addresses the decision process for coordinating energy storage plants with an electric power system over a short period of time (for example, one day). This results in a functional relationship between the coordinated schedule and the parameters of the problem. Quadratic programming techniques are suitable for the solution of this subproblem. The second subproblem employs the above described functional relationship in order to define the optimal policy in coordinating energy storage plants over an indefinite time period of operation. Dynamic programming is employed for the solution of the second subproblem.

Applications (pumped hydro storage stations, battery stations, etc.) are described. Results of a pumped hydro station application are cited.

INTRODUCTION

The cost and scarcity of high grade energy sources has created the economic attractiveness of energy storage plants and their interconnection to electric power systems. Energy storage plants, such as pumped storage hydro, and battery stations have been constructed and placed in commercial operation. Others, such as super cooled inductors, fuel cells, pumped gas, hydrogen cycle, black box, etc., are being investigated to determine feasibility and economics.

The economic benefits effected by energy storage plants result from the different production cost of electricity during different hours of the day. Low cost electric energy is used to store energy in an energy storage device. This energy is released later on to replace higher cost electric energy. The stored energy may be in a form other than electricity, for example, hydroenergy, pressurized gas, etc. The cost effectiveness of a particular energy storage plant, pumped hydro storage, has been established. The scarcity and high cost escalation of oil will further improve the economic attractiveness of energy storage plants.

The problem of optimal coordination of energy storage plants with the electric power system aims primarily at the maximization of the economic benefits, or the minimization of the overall production cost. This problem is a large and complex problem which has been approached in different ways [2], [3], [4], and [5]. This paper presents a new formulation of the problem and a decomposition technique which is computationally attractive and suitable for real time control of energy storage plants. Only the deterministic case is discussed because of space limitations.

FORMULATION

The problem of optimal coordination of energy

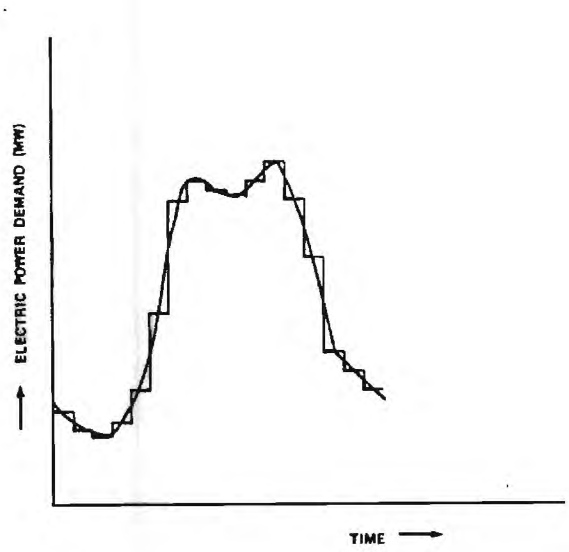


Figure 1 Discretization of the Chronological Electric Load Curve

storage devices, over a time period T, with the integrated electric power system aims primarily at the minimization of the overall system production cost. Constraints of energy availability, equipment capacities and electric power demand are imposed. The point of departure in formulating the problem is the chronological system load curve which is discretized as in Figure 1. The time period is divided into N subintervals of small duration (less than an hour). It is then assumed that the total system load remains constant for the duration of a subinterval. If x_i and y_i are the decision variables in subinterval i, defined as:

- x_i charge level (mw) of energy storage device, subinterval i
- y_i discharge level (mw) of energy storage device, subinterval i

then the impact of the control variables in the overall production cost of the system can be approximated with a quadratic function

$$f_i(x_i,y_i,L_i, G_A) = c_i^c x_i + d_i^c x_i^2 - c_i^d y_i + d_i^d y_i^2 \qquad (1)$$

where

$$c_i^d, d_i^c, d_i^d \geq 0$$

are constants depending on total system load L_i and characteristics of available generating units, G_A. In addition certain constraints apply. Let S_i^c be the set of constraints for subinterval i. Then

$$S_i^c = \begin{cases} E_{min} \le E_{i-1} + a_i x_i - b_i y_i + e_i - \ell_i \le E_{max} & (2) \\ \underline{x}_i \le x_i \le \bar{x}_i \quad \text{or} \quad x_i = 0 & (3) \\ \underline{y}_i \le y_i \le \bar{y}_i \quad \text{or} \quad y_i = 0 & (4) \end{cases}$$

where

E_{min}, E_{max}	:	Minimum and maximum capacity of the energy storage device
$\underline{x}_i, \bar{x}_i, \underline{y}_i, \bar{y}_i$	:	operating limits on charging and discharging equipment
a_i, b_i	:	constants related to the efficiency of the device
e_i	:	externally available energy (such as run of the river for a hydro plant)
ℓ_i	:	losses in subinterval i
E_{i-1}	:	energy stored in device at the end of subinterval i-1.

With above definitions the problem of optimal coordination of energy storage devices with the electric power system can be stated as follows

$$\text{Minimize} \quad x_o = \sum_{i=1}^{n} f_i(x_i, y_i, L_i, G_A) \tag{5}$$

$$\text{subject to} \quad E_o = C \tag{6}$$

$$\text{and constraint set } S_i^c, \; i = 1,2,\ldots,N \tag{7}$$

The formulated problem is rather general. It applies to any energy storage device which can be coordinated with the electric power system with single or multiple reservoirs. It also applies to the coordination of conventional hydroelectric plants with the power system. Unfortunately, the dimensionality of the model as well as the fact that certain external parameters are random variables, such as system demand L_i, externally available energy e_i, and available generators G_A, prohibit a direct solution. Fortunately, the model possesses certain properties which enable the decomposition of the problem into smaller problems, easily solvable. In the subsequent section, the decomposition technique is discussed.

Decomposition

The model possesses the following properties:

--The objective function is separable

--The constraints are also separable.

A decomposition can be effected as follows: Consider the time period T which has been divided into N intervals. Assume the period T divided into K subperiods. A subperiod is defined with the set I_k of intervals:

$$I_k = \{i \; ; \; i = n_{k+1}+1, \ldots, n_k\}$$
$$n_o = 1$$
$$k = 1,2,\ldots,K \tag{8}$$

Consider the problem

$$\text{Minimize} \quad \Lambda_\lambda = \sum_{i=1}^{n_\lambda} f_i(x_i, y_i, L_i, G_A) \tag{9}$$

$$\text{subject to} \quad E_o = c = \xi_o \tag{10}$$

$$\text{constraint set } S_i^c, \; i \epsilon I_k, \; k = 1,2,\ldots,\lambda \tag{11}$$

$$\text{and} \quad E_{n\lambda} = \xi\lambda \tag{12}$$

Let the optimal solution to above problem be

$$\Lambda_\lambda^*(\xi_o, \xi_\lambda)$$

Then the problem is formulated as a dynamic problem with the following recurrence formula

$$\Lambda_{\lambda+1}^*(\xi_o, \xi_{\lambda+1}) = \min \{\Lambda_\lambda^*(\xi_o, \xi_\lambda) + \sum_{i \epsilon I_{\lambda+1}} f_i(x_i, y_i, L_i, G_A) \tag{13}$$

$$\text{subject to} \quad E_{n\lambda} = \xi_\lambda \tag{14}$$

$$\text{constraint set } S_i^c, \; i \epsilon I_{\lambda+1} \tag{15}$$

$$\text{and} \quad E_{n_{\lambda+1}} = \xi_{\lambda+1} \tag{16}$$

The solution to the above problem is obtained as follows: The variable ξ_λ is discretized into m values, $\xi_{\lambda i}$, i = 1, 2,..., m. For a given value $\xi_{\lambda i}$ the problem defined with (13), (14), (15), and (16) collapses into an optimization problem of the quadratic programming variety. Thus, in order to obtain the solution $\Lambda_{\lambda+1}^*(\xi_o, \xi_{\lambda+1})$, the problem defined with (13), (14), (15), and (16) is solved for every value $\xi_{\lambda i}$, i = 1, 2,..., m. The solution with the least cost will be the optimum.

In summary, application of the recurrence formula (13) requires the solution of a series of quadratic programs. The model is flexible, since the dimension of the quadratic program can be independently selected, and suitable in handling the stochastic nature of certain problem parameters. In the subsequent section a number of details of the solution method employed is outlined.

SOLUTION METHOD

For fixed ξ_λ, $\xi_{\lambda+1}$ the problem defined with (13), (14), (15), and (16) collapses to the following quadratic program

$$\text{Minimize} \quad c^t z + z^t D z \tag{17}$$

$$\text{subject to} \quad E_{n\lambda} = \xi_\lambda \tag{18}$$

$$\text{constraint set } S_i^c, \; i \epsilon I_{\lambda+1} \tag{19}$$

$$\text{and} \quad E_{n_{\lambda+1}} = \xi_{\lambda+1} \tag{20}$$

$$\text{where} \quad z = [x_{n_{\lambda+1}} \ldots x_i \ldots x_{n_{\lambda+1}} \ldots y_i \ldots y_{n_{\lambda+1}}]$$

Application of the recurrence equation (13) requires the solution of a large number of quadratic programs such as the above. Thus, the quadratic model above deserves our attention. The practicality of the method depends on how efficiently the quadratic programs can be solved. The size of the quadratic model and the fact that the constraint set S_i^c is not convex present special problems. In order to arrive to an efficient algorithm the two problems have been approached as follows.

Non-Convex Set of Constraints

The constraint set is not convex because of the constraints $\{\underline{z}_i \leq z_i \leq \bar{z}_i \text{ or } z_i = 0\}$. A brute force approach is to consider all possible combinations of constraint sets, compute the optimal solution and then select the best solution. Instead, the following procedure is utilized: The above constraint is replaced with $\{0 \leq z_i \leq \bar{z}_i\}$ and the resulting problem is solved. If the constraint $\{z_i = 0 \text{ or } \underline{z}_i \leq z_i \leq \bar{z}_i\}$ is satisfied, the solution is acceptable. Otherwise, sensitivity analysis (which is readily available from the quadratic program) is employed to select constraint $\{z_i = 0\}$ or $\{\underline{z}_i \leq z_i \leq \bar{z}_i\}$ and the problem repeated. This approach has been proven to be efficient.

Size

The size of the quadratic model is greatly reduced using model reduction techniques. In this application the following procedure is followed:

(a) Omission of the term z^tDz reduces the problem defined with (17), (18), (19), and (20) into a linear program. Most of the constraints simply define bounds on the decision variables. The simplex method with upper bounds is utilized to solve the problem efficiently. Because the term z^tDz is small compared to c^tz, the obtained solution provides information regarding the effectiveness of the constraints (18), (19), and (20) in the final solution.

(b) A quadratic program is defined with the objective (17) and the effective or near effective constraints from (18), (19), and (20). (This information is known from (a)). This problem is solved with the simplex method for quadratic programming . To increase the efficiency of the solution method, constraints which impose bounds on the decision variables are implicitly handled in exactly the same way as in the simplex method with upper bounds.

The obtained solution is checked against the complete set of constraints. If violations occur, the reduced model is augmented with the violated constraints and the procedure repeated.

TEST RESULTS

The described two-level optimization model has been applied to the coordination of a 4000-MWHR reservoir capacity pumped storage hydro station with a given electric power system. The peak load of the system during a test week is 12100 MW. The generating capability of the plant is 400 MW while the pumping capability is 600 MW. Figure 2 shows a typical optimal schedule over a period of one week. Subperiods equal to the duration of one day were used in the decomposition procedure. The model provides the following additional information not shown in Figure 2.

- Optimal water level in the reservoir
- Electric power system fuel requirements
- Sensitivity of economic benefits to schedule changes
- Sensitivity of economic benefits to system parameters such as reservior capacity, generating equipment capacity, and pumping equipment capacity.

CONCLUSIONS

A decomposition technique has been described which

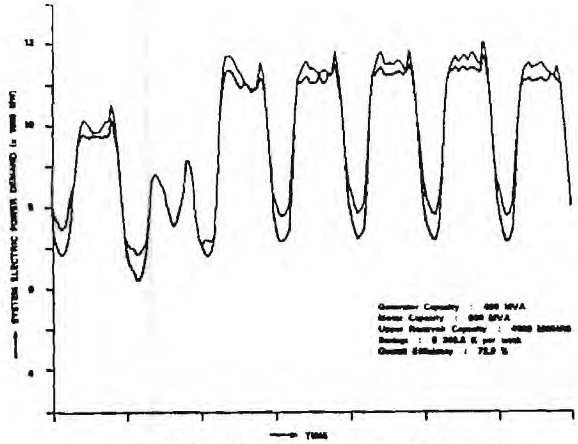


Figure 2 Chronological Electric Load and Optimal Electric Power Generation Curve

reduces the complexity of the problem of coordinating energy storage plants with an integrated electric power system. The technique results in a two level optimization procedure. At the first level quadratic programming techniques are employed to optimize plant operation over defined subperiods. At the second level dynamic programming techniques are employed to optimize plant energy storage levels at each subperiod. The described model allows exploitation of special problem structures which results in practical computational algorithms. Also, sensitivity analysis allows the optimization of plant parameters.

ACKNOWLEDGEMENTS

The work reported in this paper is supported by NSF Grant No. ENG-7816496.

REFERENCES

1. P. Wolfe, "The Simplex Method for Quadratic Programming", Econometrica, Vol. 27, 3, July 1959, pp. 382-398.

2. J. D. C. Little, "The Use of Storage Water in a Hydroelectric System", Journal of the Operations Research Society of America, Vol. III, 1955, pp. 187-197.

3. P. J. Bernard, J. F. Dopazo, and G. W. Stagg, "A Method of Economic Scheduling of a Combined Hydro and Steam Generating System", IEEE Transaction on PAS, Vol, 83, January 1964, pp. 23-30.

4. C. W. Watchorn, "Coordination of Hydro and Steam Generation", AIEE Transactions, Vol. 74, pt. III, April 1955, pp. 142-150.

5. F. A. Vizamontes and H. B. Hamilton, "Scheduling of Hydro Electric Storage Plants Using Dynamic Programming", paper presented at the IEEE PES Summer Meeting, Mexico City, Mexico, July 17-22, 1977.

6. A. P. Meliopoulos and N. Skordilis, "Coordination of Pumped Storage Stations With the Integrated Electric Power System", Proceedings of the 1979 Control of Power Systems Conference, pp. 127-131, College Station, Texas, March 19-21, 1979.

NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550

FINAL PROJECT REPORT
NSF FORM 98A

E-21-651

PLEASE READ INSTRUCTIONS ON REVERSE BEFORE COMPLETING

PART I—PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program	3. NSF Award Number
Georgia Institute of Technology 225 North Avenue, N.W. Atlanta, GA 30332	Electric Sciences and Analysis	ENG-7816496
	4. Award Period From 1/12/78 To 28/2/81	5. Cumulative Award Amount $46,293.00

6. Project Title
Optimal Coordination of Energy Storage Facilities With an Integrated Electric Power System

PART II—SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

A study of the optimal coordination of energy generating and energy storage facilities has been carried out. The optimal control of energy storage devices in an integrated electric power system has been investigated. The uncertainty associated with this control problem, its sources and impact have been analyzed. Methods of optimizing design parameters as well as operational practices of energy storage facilities, while meeting the user load demand and other constraints, have been developed. The electric power system is represented with stochastic models of alternative energy generating units, along with their fixed and incremental production costs. Also electric power demand is represented with stochastic models. A stochastic optimization method, consisting of a novel combination of nonlinear mathematical programming and dynamic programming, has been developed. The results of the investigation indicate: (a) The uncertainty associated with the operation of electric power systems impacts drastically on optimal coordinating policies of energy storage facilities with the electric power system. (b) For every electric power system there is an optimal capacity of energy storage devices. Beyond this optimal capacity, addition of energy storage facilities is economically and ecologically undesirable.

PART III—TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRA[M] Check (✓)	Approx. Date
a. Abstracts of Theses					
b. Publication Citations					
c. Data on Scientific Collaborators					
d. Information on Inventions					
e. Technical Description of Project and Results					
f. Other *(specify)*					

2. Principal Investigator/Project Director Name *(Typed)*	3. Principal Investigator/Project Director Signature	4. Date
A. P. Meliopoulos, EE		20/3/81

Grant ENG – 7816496

OPTIMAL COORDINATION OF ENERGY STORAGE FACILITIES WITH AN INTEGRATED ELECTRIC POWER SYSTEM

By

A. P. Meliopoulos

Performed for

NATIONAL SCIENCE FOUNDATION
ENGINEERING DIVISION
ELECTRICAL SCIENCES AND ANALYSIS SECTION
WASHINGTON, D.C. 20550

FINAL REPORT FOR PERIOD 1 DECEMBER 1978 TO 28 FEBRUARY 1981

FEBRUARY 1981

GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL ENGINEERING
ATLANTA, GEORGIA 30332

1981

School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

FINAL REPORT

PROJECT NO. E21-651

OPTIMAL COORDINATION OF ENERGY STORAGE FACILITIES WITH AN INTEGRATED ELECTRIC POWER SYSTEM

by

A. P. Meliopoulos

RESEARCH GRANT ENG - 7816496

1 December 1978 to 28 February 1981

Performed for

NATIONAL SCIENCE FOUNDATION
Engineering Division
Electrical Sciences and Analysis Section
Washington, D.C. 20550

TABLE OF CONTENTS

I. Abstract

A research program was undertaken with the overall objective the analysis and development of efficient models for the optimal coordination of energy storage devices with an integrated hydrothermal electric power system. The coordination problem has been studied with a stochastic model and a two level optimization procedure which has been developed specifically for this problem. The results of this research project provide useful insight to the problems of expansion planning of electric power systems and operations planning as related to storage plants. In addition the research project resulted in efficient methodologies and associated computer programs for projecting the impact of energy storage devices on overall power system costs.

The main conclusions of the research project are:

1. The uncertainty associated with the operations of electric power systems impacts drastically on optimal coordinating policies of energy storage devices with the electric power system. Thus deterministic models for scheduling energy storage devices are at best inaccurate.

2. For every electric power system there is an optimal capacity of energy storage devices. Beyond this optimal capacity, addition of energy storage devices is economically undesirable.

Above results, the models developed and the analysis procedures employed are presented in this report.

II. RESEARCH PROJECT SUMMARY

National Science Foundation Grant No. ENG - 7816496

OPTIMAL COORDINATION OF ENERGY STORAGE FACILITIES WITH AN INTEGRATED ELECTRIC POWER SYSTEM

The overall objective of the research project was the analysis and development of efficient models of the coordination of energy storage devices with an integrated hydrothermal electric power system. Such models are necessary for both expansion planning of electric power systems and operations planning. The ever increasing operating costs of electric power systems make energy storage devices economically attractive. Many different types of energy storage devices have been proposed. One type of energy storage devices, namely pumped hydrostorage plants, have proven to reduce overall cost of electric power systems. In the future other types may also become economical. This research program resulted in efficient methodologies for projecting the impact of such devices on overall system costs. In addition efficient methodologies for the optimal on-line control of energy storage devices have been generated.

This report summarizes the achievements of the research project. It is organized as follows. Section IV completely defines the coordination problem between energy storage devices and integrated electric power systems. Then several technical papers are listed which present results and more insight into the coordination problem. Throughout the report the stochastic nature of the coordination problem and the substantial impact of random events on the optimal operating policies is emphasized. In addition analysis of the problem indicates that for every integrated electric power system there is an optimal capacity of energy storage devices. Addition of energy storage devices beyond this capacity has adverse economic effects on the system. This point is illustrated and emphasized in the technical papers.

Part a of Section IV presents a formulation of the problem which leads to efficient computational procedures. Specifically the problem of coordinating an energy storage facility with an electric power system has been formulated as an optimal stochastic control problem. The objective of this problem is the minimization of the expected value of the restitution cost. The restitution cost is defined as the cost of energy stored during the charging cycle of the facility minus the energy replacement cost during the discharging cycle. The restitution cost depends on system electric load, available generating units and generating unit operating cost. Electric load and generating units are modeled as stochastic processes. The stochastic models of the electric load and generation system are given in Parts b and c of Section IV. Dynamic programming techniques have been applied to the optimal stochastic control problem. These techniques led to recurrence formulae which are of the quadratic programming variety.

The problem of computational efficiency has been addressed with respect to the solution of these recurrence formulae. The recurrence formulae are efficiently solved with a sparcity coded quadratic programming algorithm based on the dual simplex procedure. This algorithm is described in the attached technical paper "Quadratic Programming With Bounded Variables and Sparcity Techniques",

The computational complexity of the problem necessitated the investigation of alternative policies for the optimal coordination of energy storage facilities with an electric power system using the developed models. The following approaches to the solution of the optimal stochastic control problem were investigated:

a) Deterministic Approach

b) Certainty Equivalent Approach

c) Pure Open Loop Approach

d) Pure Open Loop Feedback Approach

e) Pure Feedback Approach

The result of this investigation are summarized as follows: The deterministic approach substantially underestimates the effectiveness of energy storage devices. Approaches (c), (d), and (e) result in similar operating policies and storage device effectiveness. The certainty equivalent approach provides results between the deterministic approach and any of the approached (c), (d), and (e). The computational requirement increases as we go from approach (a) to approach (e). Additional comments and results are given in the attached technical paper "Optimal Coordinating Policies of Pumped Hydrostorage Plants in the Presence of Uncertainty".

The investigations were carried out with a number of computer programs which have been designed and developed for the needs of this research project. These computer programs are available upon request. One of these computer programs performs a stochastic simulation of the operation of an electric power system. Because of the generality of this computer program and the wide range of applications it spans, it was converted into an interactive computer program and it is, currently used in the graduate courses at Georgia Tech. The accompanied technical paper "Computer Aided Instruction of Energy Source Utilization Problems" which will appear in the IEEE Transactions on Education describes the mode of utilization of this program.

III. RELATED APPLICATIONS

The concept of coordination of various devices in an electric power system is essential for the economic operation of the system. Uncertainty associated with the mode of operation of electric power system devices renders the problem stochastic nature. In addition electric load demand is also a stochastic process. Thus stochastic models are necessary for the solution of the coordination problem. The ever increasing operating costs of electric power systems make the coordination problem more important than ever. The stochastic models and procedures developed with this research project have many general characteristics and are applicable to numerous other efficient production problems under uncertainty. By limiting the scope to electric power systems we can reference the most important.

1. Dispersed Generation and Storage Systems. Electric Power Distribution Systems of the future will have distributed generation and storage capability. The analysis of such systems can be only performed in a stochastic framework. The methodologies described in this report are directly applicable to this problem.

2. Real Time Control of Hydro Plants. The operation of many hydro plants and hydrostorage plants is presently controlled with process computer. In the future all generation and storage plants will be operated with process computer. The developed models and analysis procedures described in this report can be directly implemented in these computers to optimize the operation of energy storage devices in a real time environment.

3. Load Management. These models are also suitable for investigating the effectiveness of any proposed load management scheme.

Research into the operating procedures of electric power systems significantly contributes to the optimization of the electric energy generation and supply process. The problem is important because a small improvement in the operating efficiency of the system can mean savings of millions of dollars. The recent development of energy control centers certainly is an effort to the right direction. Much work is still needed in power system analysis and optimization techniques in order to enhance the capabilities and the effectiveness of energy control centers.

IV. MODELS FOR THE COORDINATION PROBLEM

a. The Optimal Coordination Problem

This section describes the formulation of the problem of optimal coordination of energy storage facilities with an electric power system and a decomposition technique of the problem into a series of smaller dimension problems. The formulation results in a stochastic optimal control problem. Application of the decomposition technique (dynamic programming) results in a recurrence formula of the quadratic programming variety. Subsequent paragraphs describe the formulation of the problem and the decomposition technique.

Formulation

The problem of optimal coordination of energy storage devices, over a time period T, with the integrated electric power system aims primarily at the minimization of the overall system production cost. Constraints of energy availability, equipment capacities and electric power demand are imposed. The point of departure in formulating the problem is the chronological system load curve which is discretized as in Figure A.1. The time period is divided into N intervals of small duration (less than an hour). It is then assumed that the total system load remains constant for the duration of an interval. If x_i and y_i are the decision variables in interval i, defined as:

x_i charge level (mw) of energy storage device, interval i

y_i discharge level (mw) of energy storage device, interval i

then the impact of the control variables in the overall production cost of the system can be approximated with a quadratic function

$$f_i(x_i,y_i,L_i,S_{Ai}) = c_i^c x_i + d_i^c x_i^2 - c_i^d y_i + d_i^d y_i^2 \qquad (A.1)$$

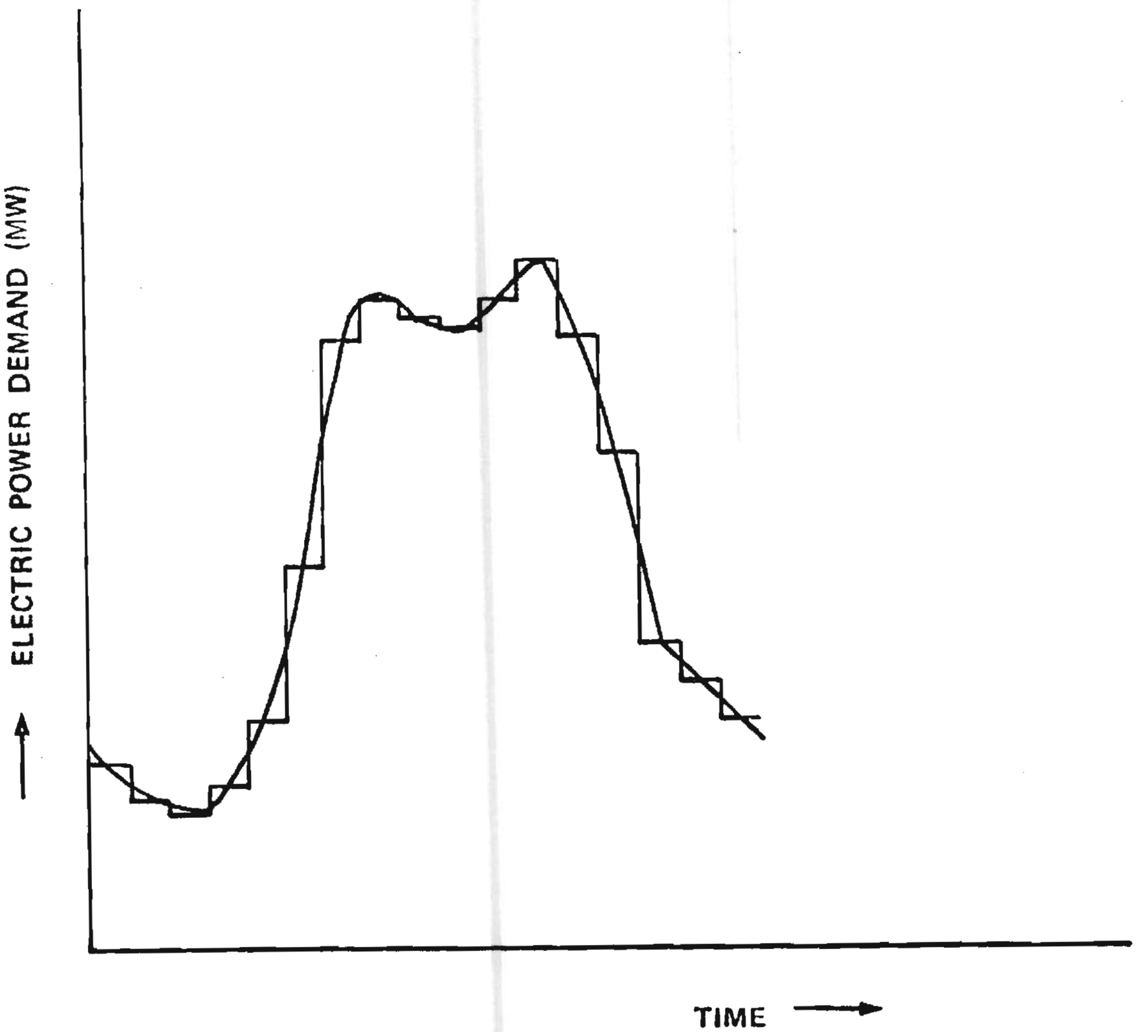


Figure A.1 Discretization of the Chronological Electric Load Curve

where

$$c_i^c, c_i^d, d_i^c, d_i^d \geq 0$$

are constants depending on total system load L_i and characteristics of available generating units, S_{Ai}. In addition certain constraints apply. Let S_i^c be the set of constraints for interval i. Then

$$S_i^c = \begin{cases} E_i = E_{i-1} + a_i x_i - b_i y_i + e_i - \ell_i \\ E_{min} \leq E_i \leq E_{max} \\ \underline{x}_i \leq x_i \leq \overline{x}_i \text{ or } x_i = 0 \\ \underline{y}_i \leq y_i \leq \overline{y}_i \text{ or } y_i = 0 \end{cases} \tag{A.2}$$

where

E_{min}, E_{max}	:	Minimum and maximum capacity of the energy storage device
$\underline{x}_i, \overline{x}_i, \underline{y}_i, \overline{y}_i$	:	operating limits on charging and discharging equipment
a_i, b_i	:	constants related to the efficiency of the device
e_i	:	externally available energy (such as run of the river for a hydro plant)
ℓ_i	:	losses in interval i
E_{i-1}	:	energy stored in device at the end of interval i-1.

At this point it should be pointed out that uncertainty is associated with the load L_i of the system and the set of available units S_{Ai}. Uncertainty is also associated with the availability of the energy storage device itself. The uncertainty is modeled as follows: An autoregressive model is developed and identified for the system load. This model is described in Part b. The uncertainty associated with 'ith the set S_{Ai} is modeled as follows: A Markov model is assumed for the individual generating units of the system. Then the statistics of the set S_{Ai} can be developed. This model is described in Part c. This model is also useful for the computation of the parameters c_i^c, c_i^d, d_i^c, d_i^d. These parameters depend on the stochastic processes L_i and S_{Ai}.

With above definitions the problem of optimal stochastic coordination of energy storage devices with the electric power system can be stated as follows:

$$\text{Minimize } J = E \sum_{i=1}^{N} f_i(x_i,y_i,L_i,S_{Ai}) \tag{A.3}$$

$$\text{subject to } E_N = C \tag{A.4}$$

$$\text{and constraint set } S_i^c,\ i = 1,2,\ldots,N \tag{A.5}$$

where S_i^c is defined with (A.2)

The quantity $\sum_{i=1}^{N} f_i(x_i,y_i,L_i,S_{Ai})$ is called the restitution cost because it equals the cost of energy stored during the charging cycles minus the energy replacement cost during the discharging cycles. The expectation E is taken with respect to the stochastic processes L_i, S_{Ai}. In above formulation the terminal condition $E_N = c$ is assumed. This is not restrictive. An initial condition $E_o = c_o$ may be assumed or none, as well. The terminal condition is assumed for practical reasons.

The formulated problem is rather general. It applies to any energy storage device which can be coordinated with the electric power system with single or multiple reservoirs. It also applies to the coordination of conventional hydroelectric plants with the power system. Because of the large dimensionality of the model and its stochastic nature, a direct solution is not feasible. The model however possesses certain properties which enable the decomposition of the problem into smaller problems, easily solvable. The decomposition leads naturally to a dynamic program. The associated recurrence equation can be solved via quadratic programming. In the subsequent section, the decomposition technique is discussed.

Decomposition

The model possesses the following properties:

* the objective function is separable
* the constraints are also separable

For this model the following decomposition scheme is natural: Consider the time period T which has been divided into N intervals. Assume the period T is divided into K subperiods. A subperiod is defined with the set I_k of successive intervals:

$$I_k=\{i;\ i = \eta_{k-1} + 1,\dots,\eta_k\} \tag{A.6}$$

$$k = 1,2,\dots,K$$

$$\eta_o = 1$$

$$\eta_K = N$$

Next define the following subproblem:

$$\text{Minimize } J_\lambda = E\left\{\sum_{i=\eta_{\lambda-1}+1}^{N} f_i(x_i,y_i,L_i,S_{Ai})\right\} \tag{A.7}$$

$$\text{subject to } E_N = c = \xi_K \tag{A.8}$$

$$\text{and constraint set } S_i^c,\ i = \eta_{\lambda-1}+1,\ldots,N \tag{A.9}$$

$$\text{and parameter } \xi_\lambda = E_{\eta_{\lambda-1}}$$

Let the optimal solution to this problem be

$$\Lambda^*_\lambda(\xi_\lambda,\xi_K)$$

Clearly the optimal solution is a function of the terminal conditions ξ_λ,ξ_K.

Applying dynamic programming techniques the following recurrence formula can be developed:

$$\Lambda^*_\lambda(\xi_\lambda,\xi_K) = \min_{\xi_{\lambda+1}} E\left\{\Lambda^*_{\lambda+1}(\xi_{\lambda+1},\xi_K) + \sum_{i\varepsilon I_\lambda} f_i(x_i,y_i,L_i,S_{Ai})\right\} \tag{A.10}$$

subject to

$$\text{constraint set } S_i^c,\ i\varepsilon I_\lambda \tag{A.11}$$

$$\text{and parameter } E_{\eta_{\lambda-1}} = \xi_\lambda \tag{A.12}$$

The expectation in the above problem is taken with respect to L_i and S_{Ai}.

The solution to the above problem is obtained as follows: For a fixed value of the variable $\xi_{\lambda+1}$ the problem defined with the recurrence formula (A.10), and constraints (A.11) and (A.12) collapses to an optimization problem of the quadratic programming variety. This problem is further examined in the attached paper: "Quadratic Programming with Bounded Variables and Sparcity Techniques". The conclusion here is that the right hand side of the recurrence formula (A.10) is a computable functional of the parameter $\xi_{\lambda+1}$. The optimal value $\xi^*_{\lambda+1}$ can be found with a direct search or using sensitivity analysis. In any case a large number of quadratic programming problems need to be solved. For this reason emphasis was placed on the efficiency of the quadratic program. The computational efficiency problem is addressed in the mentioned technical paper.

The formulation is extended in a straightforward manner to systems with more than one energy storage device. Explicit formulae are given in the technical paper "Optimal Coordinating Policies of Pumped Hydrostorage Plants in the Presence of Uncertainty". The recurrence formulae are also of the quadratic programming variety. Successive approximations dynamic programming techniques are directly applicable to the solution of these recurrence formulae.

b. Electric Load Model

This section provides a description of the stochastic model employed for the electric load. The described model has been selected after a thorough investigation and evaluation of existing models. The model is suitable for the purposes of this research project.

Analysis of historical data of electric load, $\ell(t)$, indicates that the electric load is a nonstationary stochastic process. Modelling and identification of nonstationary stochastic processes is quite difficult. Fortunately, it has been observed that an appropriate differential of the nonstationary stochastic process $\ell(t)$ may behave as a weakly stationary stochastic process. For example

$$x(t) = \ell(t) - \ell(t-2).$$

Because the electric load, $\ell(t)$, exhibits daily, weekly, and seasonal periodicity it is expedient to define a transformation of $\ell(t)$ into an assumed weakly stationary stochastic process $x(t)$ as follows:

$$x(t) = \nabla^d \nabla_s^D \ell(t)$$

where

$\nabla = 1 - B$	is a nonseasonal backward difference operator
$\nabla_s = 1-B^s$	is a seasonal backward difference operator
d	is the degree of nonseasonal differencing
B	is a backward operator defined with $B\ell(t) \triangleq \ell(t-1)$
D	is the degree of seasonal differencing

A stationary stochastic process, such as $x(t)$, can be generated if white noise is supposed to be transformed by a linear filter as in Figure B.1.

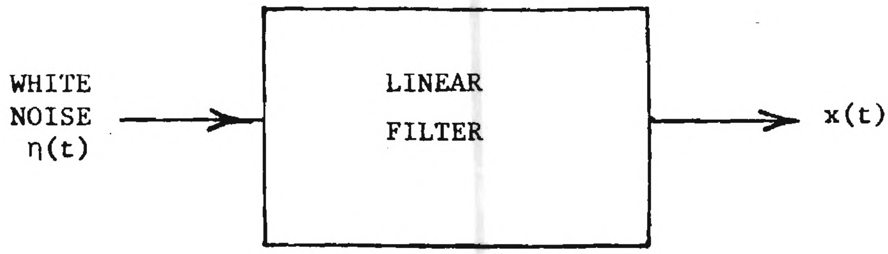


Figure B.1. Generation of a Stationary Stochastic Process

The white noise is defined as follows:

$$E[\eta(t)] = 0$$

$$\mathrm{Cov}[\eta(t)\ \eta(t+\tau)] = \begin{cases} \sigma_\eta^2 & \text{if } \tau = 0 \\ 0 & \text{otherwise} \end{cases}$$

Above models are the well known ARIMA models.

For the present application the linear filter is so selected as to result to an autoregressive model for the stochastic process $x(t)$:

$$\sum_{i=0}^{n} a_i\ x(t) + \eta(t) = 0\ , \quad a_o = 1$$

The defined model involves n parameters a_i, $i = 1, \ldots, n$, which are estimated via least square estimation. Once the model has been identified, it is evaluated to determine level of uncertainty. This is done by computing the standard deviation of the prediction.

If the level of uncertainty is acceptable, the model is complete. Otherwise another model has to be selected and the procedure is repeated. Table B.1 summarizes the procedure.

The described procedure in Table B.1 has been implemented. The results clearly show that a low order model ($n = 7$) provides good fit to historical data. The historical data used were obtained from the local utility.

Table B.1. Development of a Load Model

Step 1. Collect Load Historical Data $\ell(t)$, $t = -m, -m+1, \ldots, 0$

Step 2. Select the Order and Parameters of the Model:

$$d, D, s, \text{ and } n$$

Step 3. Estimate the Parameters $a_1, a_2, \ldots, a_n$ via Least Square Estimation

Step 4. Compute level of uncertainty (Standard Deviation of Prediction)

If Level of Uncertainty is Acceptable, stop. Otherwise select new Model Parameters (d, D, s, and n) and Go to 3.

c. Incremental Production Cost Stochastic Model

This section describes the development of a stochastic model for the computation of incremental production cost of an electric power system. The incremental production cost is essential in the optimal coordination of energy storage facilities with an electric power system.

It is shown that optimal load flows provide the incremental production cost at the busbars of energy storage facilities (lagrangian multipliers). Optimal load flows, however, are computationally unacceptable for the purposes of this research project. A good approximation is provided with the so-called busbar economic dispatch. The busbar economic dispatch provides the incremental production cost directly as a function of system load and generating unit operating costs. Because generating units are subject to forced outages a stochastic model is assumed for the generation system.

Detailed description of models follow:

Incremental Production Cost

Central to the computation of optimal policies for the operation of energy storage devices is the knowledge of electric energy production cost of the integrated electric power system. In particular it is necessary to compute the incremental cost of electric energy at the busbar of energy storage devices. This section examines the problem of computing the incremental cost of electric energy at the busbar of energy storage devices.

Consider a general electric power system as in Figure C.1. It consists of M generating units and N loads, S energy storage devices, and the interconnecting transmission system. Our objective is to determine the incremental cost of energy at busbars 1, 2, ..., s. To this purpose we start with the formulation of the operating constraints and objectives of an electric power system. The result is the so called optimal load flow which is written here in the following form

$$\text{Minimize } C_o = \sum_{i=1}^{M} f_i(u_i) \tag{C.1}$$

$$\text{subject to} \quad g(x, u, v) = 0 \tag{C.2}$$

$$h(x, u, v) \leq 0 \tag{C.3}$$

where

$f_i(u_i)$ is the operating cost of plant i, output u_i

x is the state representation of the electric power system

v is the vector of the operating level of the energy storage devices

$g(x,u,v) = 0$ load flow equations

$h(x,u,v) \leq 0$ operating constraints on system components.

The solution to this problem is secured with the Kuhn-Tucker conditions:

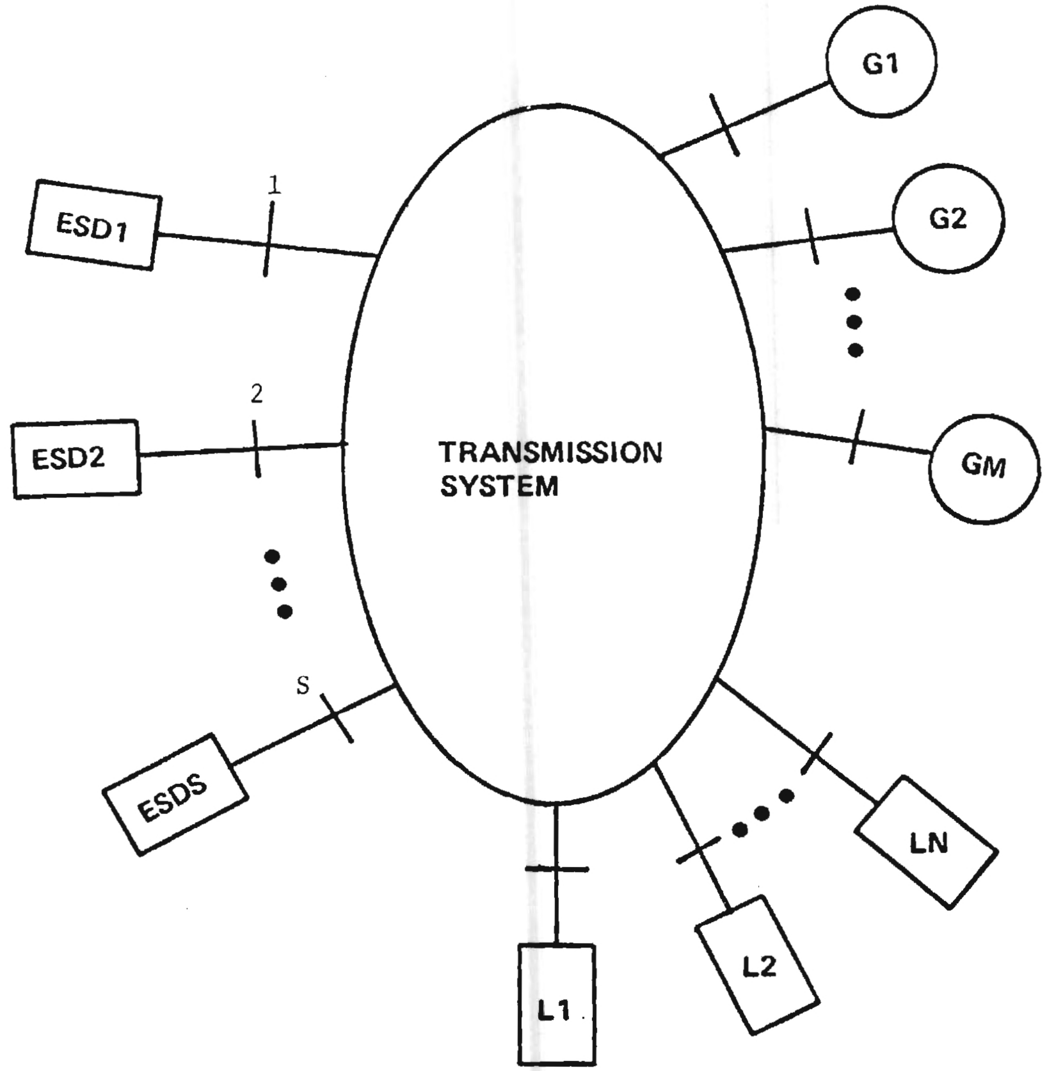


Figure C.1. An Integrated Electric Power System.

$$\mu \geq 0$$

$$\nabla C_o + \lambda^T \nabla g + \mu^T \nabla h = 0$$

$$g(x,u,v) = 0$$

$$h(x,u,v) \leq 0$$

$$\mu^T h(x,u,v) = 0$$

where ∇ is the gradient with respect to x and u.

If the load flow equations (C.2) are arranged such that the first equation expresses the real power balance at busbar 1, etc., and the Sth equation expresses the real power balance at busbar S, then the lagrangian multipliers $\lambda_1, \lambda_2, \ldots, \lambda_s$ provide the incremental cost of power at busbars 1, 2, ..., s. If $C_i(x,u,v)$ is the cost/benefit of operation of energy storage device i at level v_i, then

$$\lambda_i \triangleq \frac{\partial c_i(x,u,v)}{\partial v_i}$$

This procedure is novel. Computer programs have been developed and used routinely by the industry for operational practices. However it is impractical for planning purposes because of excessive computations and large amounts of data required. A simplification to the problem results upon the observation that actual power systems have the following features:

1) Low losses (1 to 3 percent)
2) Strong transmission systems

Feature 2) suggests that constraints (C.3) are normally inactive, and feature 1) suggests that a good approximation will be to replace the load flow equations (C.2) with a simple equation:

$$\sum_{i=1}^{M} u_i + \sum_{i=1}^{S} v_i - L - P_L = 0 \qquad \text{(C.4)}$$

$$\underline{u}_i \leq u_i \leq \overline{u}_i \quad \text{or} \quad u_i = 0 \qquad i = 1, 2, \ldots, M \qquad \text{(C.5)}$$

where

L is the total electric power load in the system

P_L is an approximate estimate of system losses (an educated guess), and it is considered constant

u_i is the output of plant i

v_i is the output of energy storage device i

Then the problem becomes

$$\text{Minimize} \quad C_o = \sum_{i=1}^{M} f_i(u_i) \qquad \text{(C.1)}$$

$$\text{subject to:} \quad \sum_{i=1}^{M} u_i + \sum_{i=1}^{S} v_i - L - P_L = 0 \qquad \text{(C.4)}$$

$$\underline{u}_i \leq u_i \leq \overline{u}_i \quad \text{or} \quad u_i = 0 \qquad i = 1, 2, \ldots, M \qquad \text{(C.5)}$$

This problem is known as the busbar economic dispatch problem. The solution of above problem is obviously a function of $L' = L - \sum_{i=1}^{S} v_i$:

$$C_o = F(L')$$

The incremental energy cost for energy storage plant i is

$$\lambda_i = \frac{\partial C_o}{\partial v_i} = \frac{\partial F(L')}{\partial v_i} = \frac{\partial F(L')}{\partial L'} \frac{\partial L'}{\partial v_i} = - \frac{\partial F(L')}{\partial L'}$$

Note that for this simplified problem

$$\lambda_i = \lambda_j \qquad i,j = 1, 2, \ldots, s$$

That is the incremental production cost is same for every energy storage device. This is a result of the assumption of constant losses in the transmission network. This assumption will not introduce an appreciable error.

In summary, given a set of generating units {1, 2, ..., M} with their cost functions $f_i(u_i)$, i = 1. 2, ... M and allowable operating limits $\underline{u}_i$, $\overline{u}_i$, i = 1, 2, ..., M, and s energy storage devices operating at levels v_i, i = 1, 2, ..., s and a total load L of the system, the total operating cost is determined from the solution of the following problem:

$$\text{Minimize} \quad C_o = \sum_{i=1}^{M} f_i(u_i) \tag{C.1}$$

$$\text{subject to:} \sum_{i=1}^{M} u_i - L' - P_L = 0 \tag{C.6}$$

$$\underline{u}_i \leq u_i \leq \overline{u}_i, \quad \text{or} \quad u_i = 0 \qquad i = 1, 2, \ldots, M \tag{C.5}$$

$$L' = L - \sum_{i=1}^{S} v_i \tag{C.7}$$

and the incremental power cost from

$$\lambda = -\frac{\partial C_o^*(L')}{\partial L'} \tag{C.8}$$

where $C_o^*(L')$ is the optimal solution of the problem defined with (C.1), (C.5), (C.6), and (C.7).

Random Outages of Generating Units

Generating units are subject to random events leading to their outages or their inability to generate electric power. Thus the availability of a generating unit is a stochastic process. Since the incremental production cost, λ, is dependent on the available generating units λ is also a stochastic process. The model of this stochastic process will be derived from a probabilistic model for the generating system. To this purpose let's consider a single generating unit consisting of m components. For a specific component define the conditional failure rate $\beta(t)$ (useful in reliability studies) as follows:

> Given a component subject to failures let $\beta(t)dt$ be the probability that the given component fails in the interval $(t, t+dt)$, assuming that it did not fail up to time t.

The probability $\beta(t)dt$ may be expressed as

$$\beta(t)dt = f(t/T \geq t)\ dt$$

where $T \geq t$ means that component x has not failed up to time t (T is a random variable equal to the time of failure).

$$\text{Now } f(t/T \geq t)\ dt = \frac{\Pr\ [\text{failure in } t,\ t+dt]}{\Pr\ [\text{failure in } T \geq t]}$$

Let $f(t)$ be the probability density function of system failure

Apparently

$$\int_{-\infty}^{\infty} f(t)\ dt = 1$$

Define the cumulative probability function

$$F(t) = \int_{-\infty}^{t} f(\tau)\ d\tau$$

Then

$$f(t/T \geq t)\, dt = \frac{f(t)\, dt}{\int_t^\infty f(\tau)\, d\tau} = \frac{\frac{dF(t)}{dt}\, dt}{1 - F(t)} = \frac{dF(t)}{1 - F(t)}$$

or

$$\beta(t)\, dt = \frac{dF(t)}{1 - F(t)}$$

Integration of above relationship from time 0 to t yields

$$F(t) = 1 - \exp\left\{ - \int_0^t \beta(\tau)\, d\tau \right\}$$

Assuming $\beta(t)$ to be constant (independent of time) we obtain

$$F(t) = 1 - \exp(-\beta t) \tag{C.9}$$

and

$$f(t) = \beta \exp(-\beta t) \tag{C.10}$$

In summary a specific component of a generating unit is subject to failures. Equations (C.9) and (C.10) describe the statistics of failures. Looking at a generating unit as a whole we observe that if a given set of components is simultaneously operational (have not failed) then the unit will be operational with maximum generation capability P. In general n(i) such states can be identified with generating unit i. There is a corresponding probability $p_i(j)$ that unit i is in state j with maximum generating capability $P_i(j)$. Thus for a specific time t the probabilistic model of a generating unit is summarized as follows:

$$Pr[A_i \leq x] = \int_{-\infty}^{x} \sum_{j=1}^{n(i)} p_i(j)\ \delta(\tau - P_i(j))\ d\tau$$

where A_i is a random variable describing the available generating capacity of unit i.

Because of failures of specific equipment and similarly because of repairs on specific failed equipment, the state of a generating unit may change in time. Define

$p_i(j,t)$ to be the probability that unit i is in state j at time t.

Then there is an associated probability that unit i will change state in a time period dt. This is illustrated in Figure C.2. Define

$\lambda_i(j,k)$ to be the conditional transition rate from state j to state k

Then

$\lambda_i(j,k)\ dt$ is the probability that the unit goes to state k in the interval (t, t + dt), assuming that it was in state j at time t.

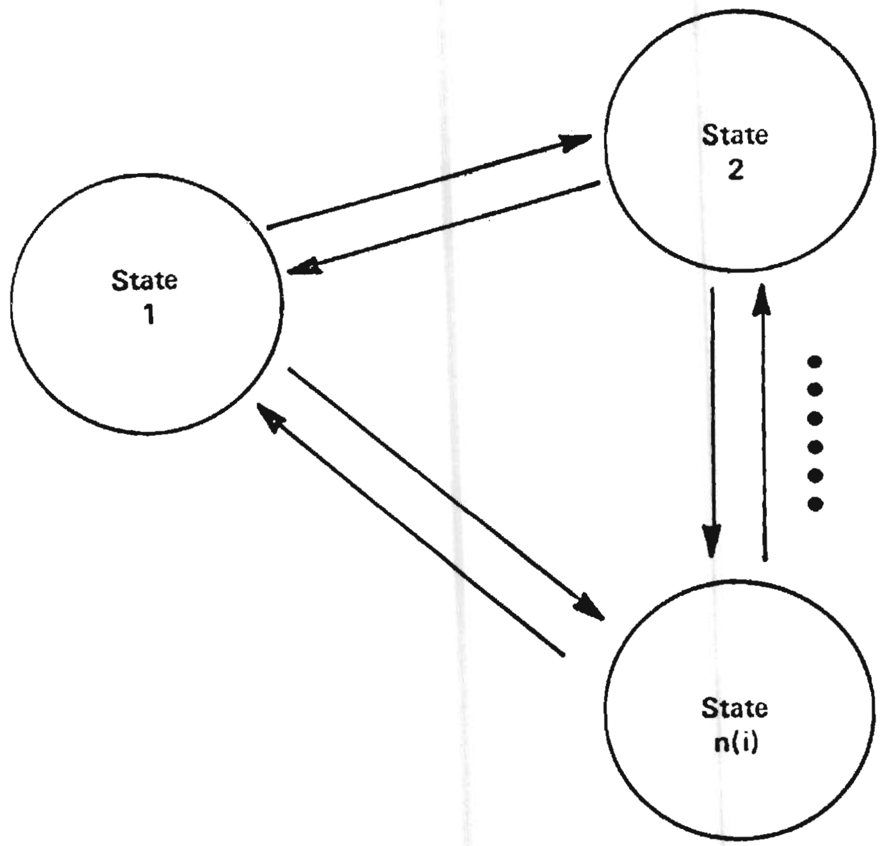


Figure C.2. Multistate Model of Availability of a Generating Unit.

With above definitions a Markov model can be generated for unit i as follows:

Assume that the probabilities

$$p_i(j,t) \qquad j = 1, 2, \ldots, n(i)$$

are known at time t. Compute the same probabilities at time t + dt. Obviously:

$$p_i(j,t+dt) = p_i(j,t)(1 - \lambda_i(j,j)dt) + \sum_{\substack{k=1 \\ k \neq j}}^{n(i)} p_i(k,t)\lambda_i(k,j)dt$$

$$j = 1, 2, \ldots, n(i)$$

By rearranging and dividing by dt we obtain

$$\frac{dp_i(j,t)}{dt} = - p_i(j,t)\lambda_i(j,j) + \sum_{\substack{k=1 \\ k \neq j}}^{n(i)} p_i(k,t)\lambda_i(k,j) \qquad \text{(C.11)}$$

In addition

$$\sum_{j=1}^{n(i)} p_i(j,t) = 1.0 \qquad \text{for any t.}$$

Given an initial condition

$$p_i(j,0), \quad j = 1, 2, \ldots, n(i) \qquad \text{(C.12)}$$

one can integrate the equations (C.11) and (C.12) to obtain p(j,t) for any j and t.

In summary a general stochastic model has been developed describing the available generating capacity of a unit.

Generating System Stochastic Model

Given a generating system with m units and their associated stochastic models:

$$\{ n(i),\ p_i(j,t),\ j = 1, \ldots, n(i),\ \ P_i(j),\ j = 1, 2, \ldots, n(i) \}$$

$$i = 1, 2, \ldots, m$$

There are

$$N = \prod_{\text{all } i} n(i)$$

discrete states of the generating system. This is a large number of states. Most of these states, however, have a low probability of existence. Thus a tremendous reduction in the number of states is achieved if it is decided to disregard all states with probability less than a threshold value p_{th}. This value may be selected to be in the order of few hundredths.

The identification of the states with probability of existence greater than p_{th} is achieved as follows.

Consider state $X_{GK}(t)$ of the generating system. This state is defined as follows:

$$X_{GK}(t) = \{p_i(j(k),t),\ i = 1, 2, \ldots, m\}$$

And the probability of existence is

$$Pr[X_{GK}] = \prod_{\text{all } i} p_i(j(k),\ t) \tag{C.13}$$

The state X_{GK} is rejected if

$$Pr[X_{GK}] < p_{th} \tag{C.14}$$

Implementation

The described model can be easily implemented. To this purpose consider a period of duration T. This period can be divided into K subperiods. Each subperiod may be several hours long. At this point the following reasonable assumption can be made: The state of the generation system may change only at the beginning of a subperiod. This assumption guarantees that the state of the generating system will not alter for the duration of one subperiod. Thus the incremental production cost during one subperiod, given the state of the generating system will be a function of system electric load only. This suggests the following computational procedure: Consider subperiod t. Using the Markov model the probabilities $p_i(j,t)$ can be computed via a numerical integration of the equations C.11) and (C.12). Then all the states of the generating system with probability greater than p_{th} can be computed using equations (C.13) and (C.14).

Finally for every state X_{GK} of the generating system with $P_r[X_{GK}] \geq p_{th}$, the incremental production cost versus system electric load can be computed using equations (C.8) after solving problem defined with (C.1), (C.5), (C.6) and (C.7). The procedure needs to be repeated for every subperiod t.

A computer program with the symbolic name NSFA has been developed for the computation of incremental production cost versus system electric load. The described Markov model of the generating system is employed. A flow chart of the program is given in Figure C.3.

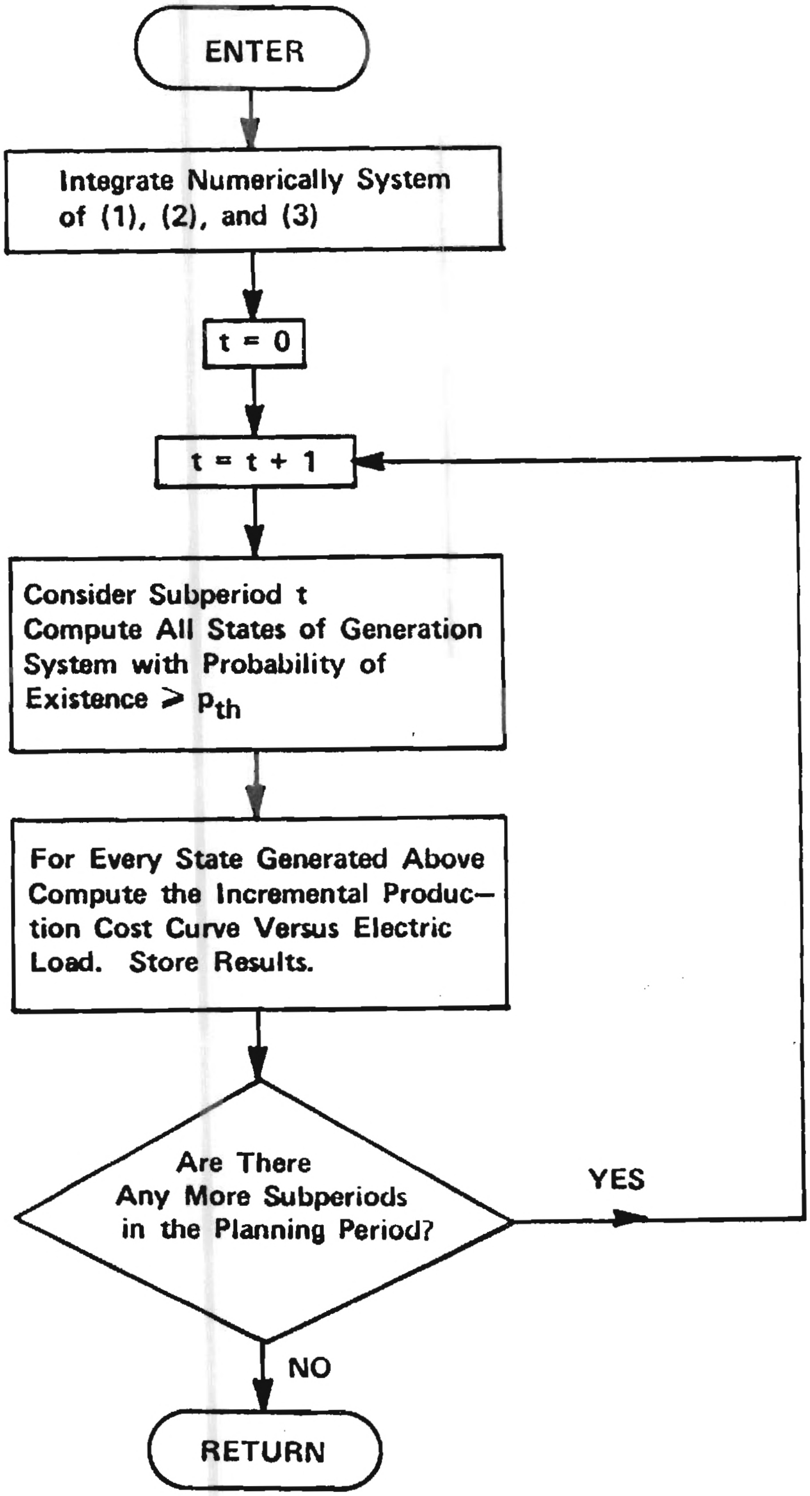


Figure C.3. Flow Chart of Computer Program NSFA.

V. OPTIMAL COORDINATION OF ENERGY STORAGE DEVICES AND ANALYSIS TECHNIQUES

COORDINATION OF PUMPED STORAGE STATIONS
WITH THE INTEGRATED ELECTRIC POWER SYSTEM

A. P. Meliopoulos and N. Skordilis
Georgia Institute of Technology
School of Electrical Engineering
Atlanta, Georgia 30332

Abstract

Pumped storage stations are examined as a tool for electric load management in the utility side. The problem of coordinating pumped storage plants with an integrated electric power system in order to maximize economic benefits is formulated as a two level optimization procedure. At the first level, a linear programming model is used to optimize the operation of storage plants for a period of one day with assumed terminal conditions, (level of energy stored, plant head, etc.) At the second level, a gradient method is employed to optimize terminal conditions for each day in a long operating period. The method is efficient and useful for operation planning as well as for long term expansion planning of electric power systems. Economic benefits resulting from optimal operation of pumped storage stations are demonstrated on a realistic 12000 MW system.

Introduction

Pumped storage hydro stations provide a practical tool for electric load management. In general, electric load management can be defined as any action taken by either a utility or a regulatory agency to influence or modify the characteristics of the electric load shape. In particular, supply management modifies the apparent load shape as seen by the generation system without affecting the real loads of individual customers. Energy storage stations, such as pumped storage hydro stations, provide the capability of supply management.

Supply management, performed with the aid of pumped storage hydro stations, incurs certain economic benefits to the particular utility. The economic benefits result from the difference in incremental production costs at different total system load levels. In order to insure maximal economic benefits, the operation of pumped storage hydro stations should be coordinated with the operation of the integrated electric power system.

The problem of coordination of pumped storage hydro stations with an electric power system has been studied [2,3,6,7] and several models have been developed. In general, these models are complex and impose heavy computational requirements.

This paper formulates the problem of optimal coordination of pumped storage hydro plants as a two level optimization problem. At the first level, a linear programming model is used to optimize the operation of storage plants for a period of one day with assumed terminal conditions, i.e. level of energy stored at the beginning of the day, available thermal generating units, etc. At the second level, a gradient method is employed to optimize terminal conditions for each day in a long operating period.

The two level optimization model is described next followed with test results as well as possible applications.

PROBLEM FORMULATION

The objective in coordinating the operation of pumped hydro stations with an integrated electric power system is the minimization of the overall operational cost. At the same time a number of constraints are imposed because of equipment and reservoir capacities. The end result is a complex and dimensionally large problem. To reduce complexity and size, it is expedient to decompose the problem into two subproblems as follows:

(a) Develop a mathematical model for the optimal scheduling of pumped hydro stations over a time period of one day by assuming:
  --the stored water level at the beginning and the end of the day is specified
  --the available thermal generating units and their cost curves are known
  --the total system load over the period is deterministically known
  --other related information, for example power import, etc., is assumed to be deterministically known for the specified period of one day.

(b) Develop a second level optimization model using the mathematical model from (a), in order to optimize initial conditions (such as stored water level at the beginning of a day). At this level, uncertainty of various parameters impacting on the optimal operation of pumped hydro stations can be incorporated. However, at the present time, only the deterministic case has been attacked.

The two subproblems are described next.

Optimal Scheduling of Pumped Hydro Stations Over a Period of One Day

Formulation

The problem of scheduling the operation of pumped hydro stations over a specified period of one day is formulated as an optimization one of the linear programming variety. The objective is defined to be the minimization of the energy restitution cost. Constraints are imposed by the size of the reservoir and the capacity of the pumps and/or the generators.

The point of departure in the development of the model is the discretization of the chronological total system load as it is shown in Figure 1. The time

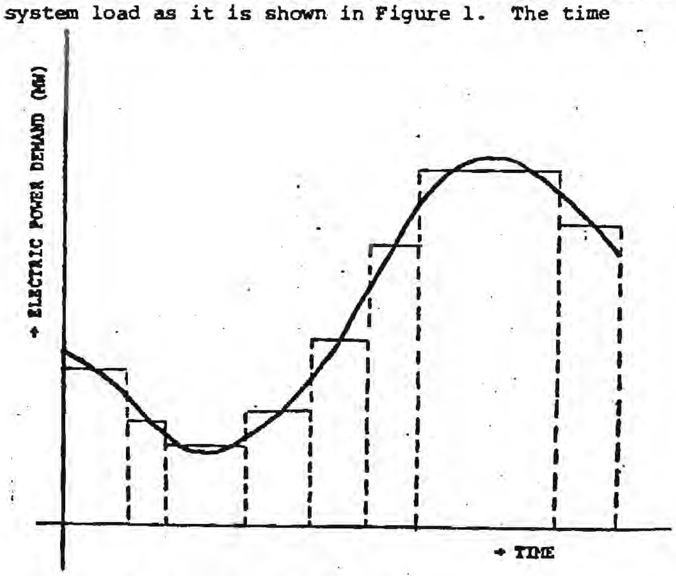


Figure 1. Discretization of the Load versus Time Curve

period of one day is divided into n intervals and the load is assumed constant throughout the duration of one interval. As the number of intervals n becomes large, this model approaches the actual chronological load curve. The discretization of the load curve allows the assumption that the state of operation of pumped storage hydro stations will remain unchanged throughout the duration of one interval. If it is assumed that during interval i, x_i (MW) are consumed by the station and during interval j, y_j (MW) are returned to the system as it is shown in Figure 2, then the problem of optimal

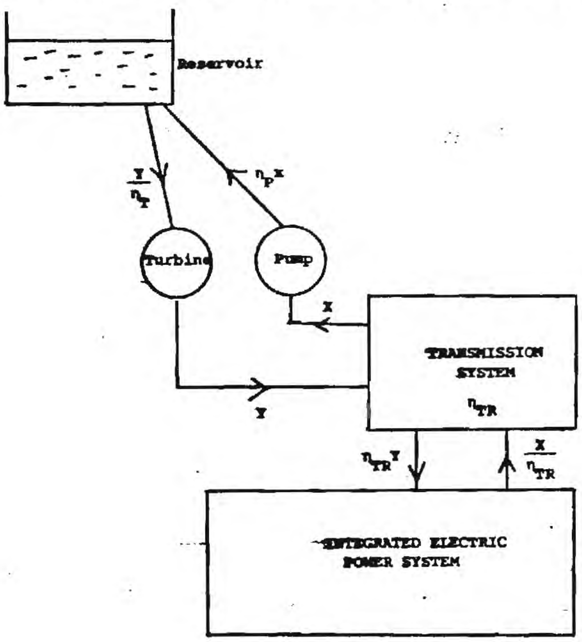


Figure 2. A Snapshot of the Operation of the System With Indicated Flow of Power

scheduling over the specified period, let it be day k, can be stated as follows:

$$\text{Minimize} \quad x_o = \sum_{i=1}^{n_k} C_i^k(L_i^k, G_A^k)\,\tau_i^k x_i - \sum_{j=1}^{n_k} C_j(L_j^k, G_A^k)\,\tau_j^k y_j \tag{1}$$

subject to

$$\varepsilon_o \le V_{k-1} + \sum_{j=1}^{m} a_j^k \tau_j^k x_j - \sum_{j=1}^{m} b_j^k \tau_j^k y_j + \sum_{j=1}^{m} s_j^k \le \varepsilon_c \tag{2}$$

$$m = 1,2,\ldots,n_k-1$$

$$V_{k-1} + \sum_{j=1}^{n_k} a_j^k \tau_j^k x_j - \sum_{j=1}^{m} b_j^k \tau_j^k y_j + \sum_{j=1}^{n_k} s_j^k = V_k \tag{3}$$

$$x_j y_j = 0 \qquad j = 1,2,\ldots,n_k \tag{4}$$

$$x_{min} \le x_j \le x_{max} \quad \text{or} \quad x_j = 0 \qquad j = 1,2,\ldots,n_k \tag{5}$$

$$y_{min} \le y_j \le y_{max} \quad \text{or} \quad y_j = 0 \qquad j = 1,2,\ldots,n_k \tag{6}$$

where

$C_j^k(L_j^k, G_A^k)$	energy replacement cost of the system at the location of the energy storage station at interval j, day k
G_A^k	set of available thermal generating units, day k
L_j^k	system electric load level, day k, interval j
τ_j^k	duration of interval j, day k
a_j^k	conversion factor (stored units/mwhr), interval j, day k, pumping cycle
b_j^k	conversion factor (stored units/mwhr), interval j, day k, generation cycle
s_j^k	energy inflow from supplementary energy source, interval j, day k
ε_c	upper reservoir capacity
ε_o	minimum allowable storage level
x_j	electric power input, interval j, day k, pumping cycle
y_j	electric power output, interval j, day k, generation cycle
V_{k-1}	amount of stored energy at the beginning of day k
x_{max}, x_{min}	maximum and minimum allowable level of electric power input, pumping cycle
y_{max}, y_{min}	maximum and minimum allowable level of electric power output, generation cycle
n_k	number of intervals in day k

Objective (1) expresses the restitution cost over the specified period. The constants $C_j^k(L_j^k, G_A^k)$ represent incremental production cost at total system load L_j^k. Constraints (2) impose upper reservoir capacity constraints. Constraint (3) imposes terminal conditions that is specified water level at the end of the specified period. Constraints (4) exclude double mode operation, i.e. simultaneous pumping and generation. Finally, constraints (5) and (6) observe the capacities of the pumping and generation equipment.

In summary, the problem of optimal coordination of pumped storage stations with an integrated electric power system is formulated as an optimization problem. Note that the optimal coordination problem over a period of one day is defined in terms of the water volume in the reservoir at the beginning and end of the day under consideration, V_{k-1}, V_k and the available generating units G_A^k.

Thus, the defined optimization problem should be viewed as a functional: Given the variables V_{k-1} and V_k and a set G_A^k, solution of the above problem will return the optimally coordinated operation of the pumped hydro station. In order to emphasize this point, we write

$$RC_k^* = f(V_{k-1}, V_k, G_A^k) \tag{7}$$

where RC_k^* is the optimal restitution cost given V_{k-1}, V_k and the set G_A^k.

Method of Solution

The defined optimization problem with the objective function (1) and constraints (2), (3), (4), (5), and (6) is rather complicated because of the nonlinear constraints (4) and the logical "or" operation involved in constraints (5) and (6). These complexities are resolved if the following assumptions are introduced prior to the solution:

- Intervals in load valley areas are allocated to pumping operation, while intervals in the peak load area are allocated to generation operation.
- The logical "or" operation in constraints (5) and

(6) is resolved by eliminating those constraints which may be ineffective in the final solution.

These assumptions transform the described optimization problem into a usual linear program. The simplex method with upper bounds is employed for its solution. It should be emphasized that once the solution has been obtained, the validity of the assumptions is checked. If the assumptions are not valid, they should be appropriately changed and the problem solved again.

In summary, the developed model determines the optimal schedule of pumped hydro stations as well as the optimal value of the energy restitution cost for any day k if the water level at the beginning and end of the day, V_{k-1} and V_k, respectively, are known.

Coordination of Water Level in the Reservoir at the Beginning of Each Day

In this section, the computational procedure to optimize the water level in the reservoir at the beginning of each day is described. The procedure to be outlined is applicable to periods of any length. However, the discussion will be confined to a period of one week. For pumped storage stations this case is of practical importance.

The point of departure is the stated objective of minimizing the restitution cost. If the period contains 7 days (a week), the restitution cost will be

$$RC = \sum_{k=1}^{7} RC_k^*$$

where RC_k^* is a functional defined by (7). Or

$$RC = \sum_{k=1}^{7} f(V_{k-1}, V_k, G_A^k).$$

Since the functional f, implicity incorporates all applicable constraints, the optimal water level in the reservoir at the beginning of each day is defined to be the solution of the following minimization problem.

Minimize

$$RC = \sum_{k=1}^{7} f(V_{k-1}, V_k, G_A^k) \qquad (8)$$

subject to

$$\varepsilon_o \le V_k \le \varepsilon_c \qquad k = 0, 1, \ldots, 7 \qquad (9)$$

Solution Method

There are many algorithmic possibilities towards a solution of the optimization problem defined with the objective (8) and constraints (9). A first order descent method has been successfully applied to this problem. At the i^{th} iteration, the terminal conditions V_k, $k = 0, 1, \ldots, 7$ are updated as follows:

$$\begin{bmatrix} V_0 \\ V_1 \\ \vdots \\ V_7 \end{bmatrix}^{(i+1)} = \begin{bmatrix} V_0 \\ V_1 \\ \vdots \\ V_7 \end{bmatrix}^{(i)} - \alpha \begin{bmatrix} \frac{\partial RC}{\partial V_0} \\ \frac{\partial RC}{\partial V_1} \\ \vdots \\ \frac{\partial RC}{\partial V_7} \end{bmatrix}$$

or

$$V^{(i+1)} = V^{(i)} - \alpha \nabla (RC).$$

Since the function RC is computed from the solution of 7 linear programs, the gradient vector $\nabla(RC)$ can be readily computed from the corresponding dual linear programs [9]. The constant α is arbitrarily selected and it is adjusted during iterations to speed up convergence. A line search is employed for these adjustments.

Test Results

The described two level optimization model has been implemented, and preliminary results with a test system consisting of 68 thermal generating units have been obtained. A listing of the units, as well as their capacities, cost functions, etc., is given in Table 1.

TABLE 1. LIST OF TEST SYSTEM GENERATING UNITS

UNIT	CAPACITY (MW)	MIN POWER (MW)	COST COEFFICIENTS A($/HOUR)	B($/MWH)	C($/MW/MWH)
1	42.00	25.00	0.1335	7.2320	0.1213
2	42.00	25.00	0.1335	7.2320	0.1213
3	45.00	25.00	0.1051	8.9242	0.0920
4	45.00	25.00	0.1051	8.9242	0.0920
5	60.00	40.00	0.3380	20.7368	0.1420
6	62.00	40.00	0.3380	20.7369	0.1420
7	63.00	40.00	0.2399	18.1937	0.0965
8	64.00	40.00	0.2399	18.1937	0.0968
9	651.00	490.00	0.6108	8.0472	0.0017
10	655.00	490.00	0.7459	8.2555	0.0013
11	831.00	530.00	1.0344	7.9055	0.0015
12	881.00	530.00	1.0344	7.9055	0.0015
13	221.00	135.00	0.2230	8.9854	0.0029
14	300.00	180.00	0.2513	8.3155	0.0035
15	481.00	270.00	0.4281	8.5736	0.0021
16	466.00	280.00	0.3821	8.4735	0.0025
17	104.00	60.00	0.1366	8.6918	0.0169
18	105.00	60.00	0.1366	8.6918	0.0168
19	90.00	55.00	0.1366	8.6918	0.0168
20	465.00	325.00	0.4289	9.2511	0.0016
21	245.00	145.00	0.1647	8.0305	0.0014
22	245.00	145.00	0.1647	8.0303	0.0014
23	50.00	35.00	0.0846	21.1315	0.0387
24	81.00	50.00	0.1504	17.7419	0.0132
25	23.00	15.00	0.2406	8.8153	0.0134
26	24.00	15.00	0.0524	11.2034	0.1167
27	161.00	95.00	0.0524	11.2034	0.1167
28	103.00	60.00	0.1710	9.4004	0.0303
29	103.00	60.00	0.1710	9.4004	0.0303
30	103.00	60.00	0.1710	9.4004	0.0303
31	135.00	80.00	0.1327	10.5389	0.0099
32	136.00	80.00	0.1327	10.5389	0.0099
33	355.00	215.00	0.3496	10.2503	0.0038
34	355.00	215.00	0.3496	10.2503	0.0035
35	15.00	15.00	0.2412	9.6997	0.8357
36	15.00	15.00	0.2412	9.6997	0.8357
37	34.00	34.00	0.5105	8.8044	0.3266
38	34.00	34.00	0.5105	8.8044	0.3265
39	38.00	38.00	0.5224	8.9242	0.3264
40	34.00	34.00	0.5085	8.7696	0.3253
41	35.00	35.00	0.5303	9.1451	0.3393
42	43.00	43.00	0.6700	8.4361	0.2274
43	44.00	44.00	0.6700	8.4361	0.2274
44	44.00	44.00	0.6700	8.4361	0.2274
45	53.00	53.00	0.6700	8.4361	0.2274
46	53.00	53.00	0.6700	8.4361	0.2274
47	54.00	54.00	0.6700	8.4361	0.2274
48	54.00	54.00	0.6700	8.4361	0.2274
49	54.00	54.00	0.6700	8.4361	0.2274
50	54.00	54.00	0.6700	8.4361	0.2274
51	37.00	37.00	0.5348	9.2591	0.3412
52	37.00	37.00	0.5348	9.2591	0.3412
53	38.00	38.00	0.5348	9.2591	0.3412
54	54.00	54.00	0.7531	9.4819	0.2556
55	55.00	55.00	0.7531	9.4819	0.2556
56	55.00	55.00	0.7531	9.4819	0.2556
57	55.00	55.00	0.7531	9.4819	0.2556
58	55.00	55.00	0.7531	9.4819	0.2556
59	55.00	55.00	0.7531	9.4819	0.2556
60	783.00	485.00	0.5457	6.4249	0.0004
61	783.00	485.00	0.5457	6.4249	0.0004
62	783.00	485.00	0.5457	6.4249	0.0004
63	783.00	485.00	0.5457	6.4249	0.0004
64	400.00	200.00	0.0000	33.5650	0.0000
65	300.00	100.00	0.0000	35.7570	0.0000
66	300.00	100.00	0.0000	35.7570	0.0000
67	300.00	100.00	0.0000	35.7570	0.0000
68	225.00	100.00	0.0000	35.7570	0.0000

Figure 3 illustrates a function $C(L, G_A)$ which represents the incremental production cost of the inte-

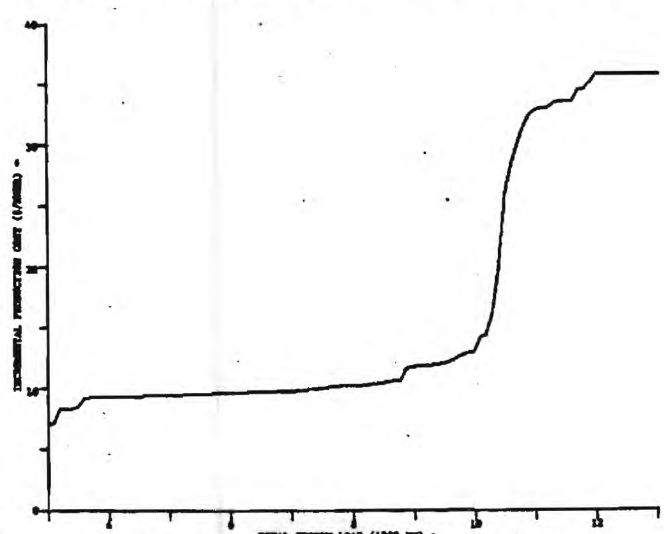


Figure 3. Incremental Production Cost Versus Total System Load.

grated electric power system. This curve is constructed from the solution of a number of economic dispatch problems at different load levels.

Historical load data from a Southeast utility have been used to obtain test results. Figure 4 shows a sample set of chronological load data for a period of

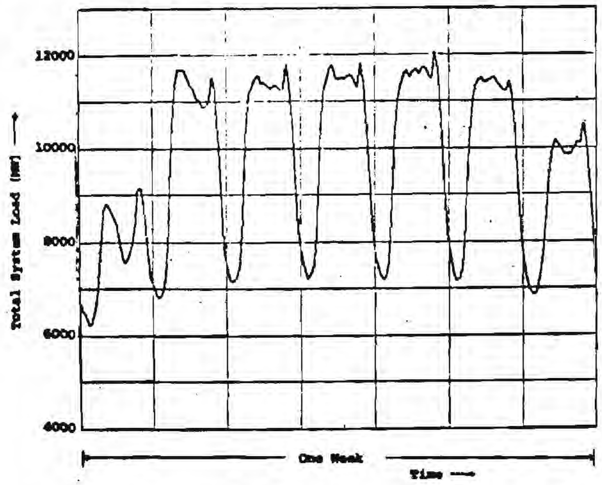


Figure 4. A Sample of Chronological Hourly Load Data for the Test System.

one week. Figure 5 shows the optimal schedule of a 200 MW pumped hydro station for the load data of Figure 4, and the incremental production cost of Figure 3. The

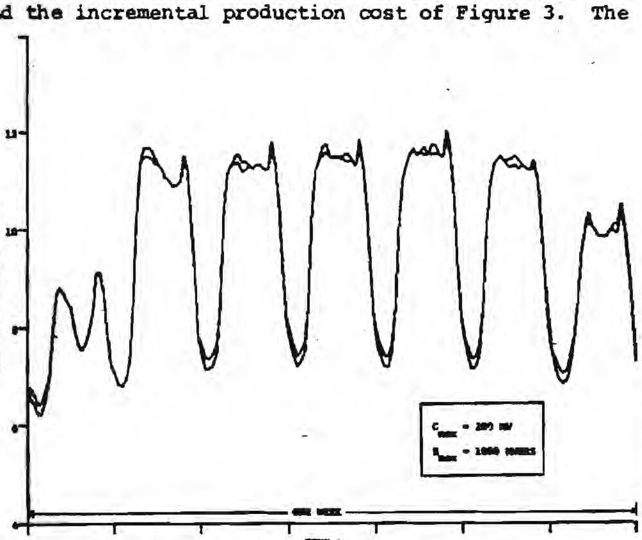


Figure 5. Optimal Schedule of a Pumped Hydro Station Superimposed on the Chronological Load Curve.

schedule of the pumped hydro station is superimposed on the chronological load curve. As it should be expected the optimal schedule is such as to "shave" the daily peaks while "filling" the valleys of the load.

The developed method can be used for parametric studies relating the savings resulting from operation of the pumped hydro station to design parameters such as capacity of generators, pumps as well as reservoir capacity. Preliminary results in this context are given in Figure 6. A number of conclusions can be drawn from Figure 6. The capacity of the generating units of a pumped hydro station limits the realizable savings beyond a certain reservoir capacity. Also, for a given generating unit capacity, there exists an optimal reservoir capacity. Thus generating unit capacity and reservoir capacity impose limits on the realizable economic benefits resulting from supply management practices using a pumped storage hydro station.

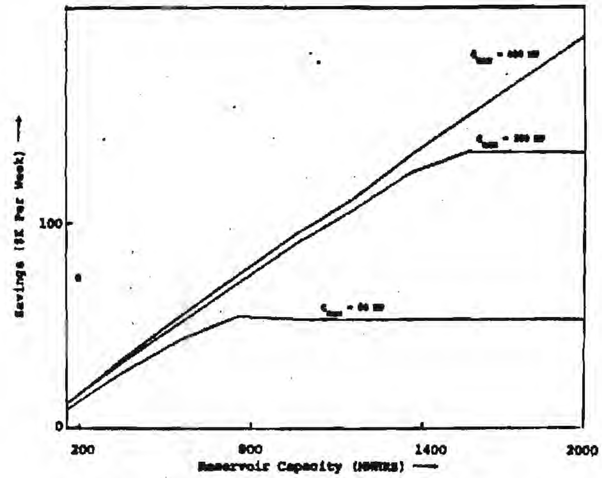


Figure 6. Savings Resulting from the Operation of a Pumped Hydro Station as a Function of Generating and Reservoir Capacity.

Conclusions and Recommendations

The problem of optimal scheduling of pumped hydro storage facilities over a cycle of one week has been formulated as a two level deterministic optimization process. At the low level the optimal operation of a pumped storage facility is formulated as a linear programming problem which is solved with the simplex method with upper bounds. Terminal conditions are assumed fixed. At the second level, terminal conditions are optimized over a period of one week with a gradient technique.

The method is computationally efficient and a useful tool for assessing economic benefits resulting from coordination of pumped storage hydro stations with an integrated electric power system. The method is also applicable to other energy storage stations.

Preliminary studies indicate that the use of pumped storage hydro stations as a supply management tool results in substantial economic benefits. Depending on the system involved, these benefits may justify the investment cost required. If the upper reservoir is located in the run of a river, additional benefits are realized from utilization of the potential energy of the river water. In recent years, we have witnessed sharp escalation of high grade fuel costs and moderate escalation of low grade fuel costs. Assuming this trend continues, greater economic benefits are projected for pumped storage hydro stations. For these reasons, it is our opinion that pumped storage hydro stations are practical and economically attractive supply management tools.

The approach taken is a deterministic one. Definitely, random forced outages of thermal generating units as well as uncertainty associated with the load forecast have an impact on the economic utilization of pumped hydro storage. The presented methodology can be extended to the probabilistic case. Further work in this area is required.

References

1. A. F. Glimn and L. K. Kirchmayer, "Economic Operation of Variable Head Hydroelectric Plants," IEEE Trans. Power App. Syst., vol. 77, Dec. 1958, pp. 1070-1079.

2. P. J. Bernard, J. F. Dopazo, and G. W. Stagg, "A Method of Economic Scheduling of a Combined Hydro and Steam Generating System," IEEE Trans. PAS, vol. 83, Jan. 1964, pp. 23-30.

3. G. H. McDaniel and A. F. Gabrielle, "Dispatching Pumped Storage Hydro," IEEE Trans. PAS, vol. 85,

1966, pp. 465-471.

4. Survey of Pumped Storage Projects in the United States and Canada to 1975, IEEE Committee Report, presented at the 1976 Winter Power Meeting.

5. "Hydro-Electric Power Resources of the United States: Developed and Undeveloped," Document No. 015-000-00347-2, 1976.

6. C. D. Galloway and R. J. Ringlee, "An Investigation of Pumped Storage Scheduling," IEEE Trans. PAS, vol. 85, no. 5, May 1966, pp. 459-465.

7. F. A. Viramantes and H. B. Hamilton, "Scheduling of Hydro Electric Storage Plants Using Dynamic Programming," paper presented at the IEEE PES Summer Meeting, Mexico City, Mexico, July 17-22, 1977.

8. A. P. Meliopoulos, G. J. Cokkinides and N. Skordilis, "Production Costing Methodologies," Final Report prepared for Westinghouse Electric Corporation, East Pittsburgh, PA.

9. D. G. Luenberger, Introduction to Linear and Nonlinear Programming, Addison-Wesley Publishing Corporation, 1973.

Acknowledgments

The methodologies presented in this paper have been developed by the Electric Power Research Laboratory of Georgia Tech under project E21-647 sponsored by Westinghouse Electric Corporation. The financial support as well as permission to present this paper is greatly appreciated. The authors are also indebted to Mr. John T. Day for his valuable comments and deliberations throughout the development of this project.

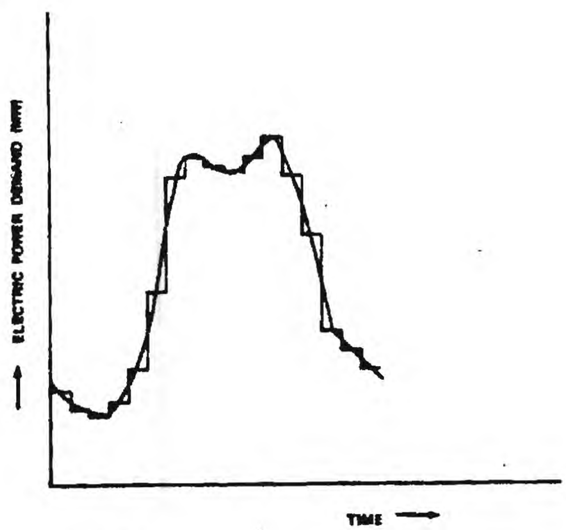
A DECOMPOSITION TECHNIQUE TO DETERMINE OPTIMAL COORDINATING POLICIES OF ENERGY STORAGE PLANTS WITH ELECTRIC POWER SYSTEM

A. P. Meliopoulos

Georgia Institute of Technology
School of Electrical Engineering
Atlanta, Georgia 30332

Abstract - The problem of coordinating energy storage plants with an integrated hydroelectric power system to achieve minimum overall operating costs is a large and complex stochastic decision problem. This paper describes a decomposition technique which reduces the complexity of the problem and results in practical computational algorithms. The method is based on the definition of two subproblems. The first subproblem addresses the decision process for coordinating energy storage plants with an electric power system over a short period of time (for example, one day). This results in a functional relationship between the coordinated schedule and the parameters of the problem. Quadratic programming techniques are suitable for the solution of this subproblem. The second subproblem employs the above described functional relationship in order to define the optimal policy in coordinating energy storage plants over an indefinite time period of operation. Dynamic programming is employed for the solution of the second subproblem.

Applications (pumped hydro storage stations, battery stations, etc.) are described. Results of a pumped hydro station application are cited.



A 1

Figure 1 Discretization of the Chronological Electric Load Curve

INTRODUCTION

The cost and scarcity of high grade energy sources has created the economic attractiveness of energy storage plants and their interconnection to electric power systems. Energy storage plants, such as pumped storage hydro, and battery stations have been constructed and placed in commercial operation. Others, such as super cooled inductors, fuel cells, pumped gas, hydrogen cycle, black box, etc., are being investigated to determine feasibility and economics.

The economic benefits effected by energy storage plants result from the different production cost of electricity during different hours of the day. Low cost electric energy is used to store energy in an energy storage device. This energy is released later to replace higher cost electric energy. The stored energy may be in a form other than electricity, for example, hydroenergy, pressurized gas, etc. The cost effectiveness of a particular energy storage plant, pumped hydro storage, has been established. The scarcity and high cost escalation of oil will further improve the economic attractiveness of energy storage plants.

The problem of optimal coordination of energy storage plants with the electric power system aims primarily at the maximization of the economic benefits, or the minimization of the overall production cost. This problem is a large and complex problem which has been approached in different ways [2], [3], [4], and [5]. This paper presents a new formulation of the problem and a decomposition technique which is computationally attractive and suitable for real time control of energy storage plants. Only the deterministic case is discussed because of space limitations.

FORMULATION

The problem of optimal coordination of energy storage devices, over a time period T, with the integrated electric power system aims primarily at the minimization of the overall system production cost. Constraints of energy availability, equipment capacities and electric power demand are imposed. The point of departure in formulating the problem is the chronological system load curve which is discretized as in Figure 1. The time period is divided into N subintervals of small duration (less than an hour). It is then assumed that the total system load remains constant for the duration of a subinterval. If x_i and y_i are the decision variables in subinterval i, defined as:

x_i charge level (mw) of energy storage device, subinterval i

y_i discharge level (mw) of energy storage device, subinterval i

then the impact of the control variables in the overall production cost of the system can be approximated with a quadratic function

$$f_i(x_i,y_i,L_i,G_A) = c_i^c x_i + d_i^c x_i^2 - c_i^d y_i + d_i^d y_i^2 \qquad (1)$$

where

$$c_i^d, d_i^c, d_i^d \geq 0$$

are constants depending on total system load L_i and characteristics of available generating units, G_A. In addition certain constraints apply. Let S_i^c be the set of constraints for subinterval i. Then

$$S_i^c = \begin{cases} E_{min} \le E_{i-1} + a_i x_i - b_i y_i + e_i - \ell_i \le E_{max} & (2) \\ \underline{x}_i \le x_i \le \bar{x}_i \quad \text{or} \quad x_i = 0 & (3) \\ \underline{y}_i \le y_i \le \bar{y}_i \quad \text{or} \quad y_i = 0 & (4) \end{cases}$$

where

E_{min}, E_{max}	:	Minimum and maximum capacity of the energy storage device
$\underline{x}_i, \bar{x}_i, \underline{y}_i, \bar{y}_i$	:	operating limits on charging and discharging equipment
a_i, b_i	:	constants related to the efficiency of the device
e_i	:	externally available energy (such as run of the river for a hydro plant)
ℓ_i	:	losses in subinterval i
E_{i-1}	:	energy stored in device at the end of subinterval i-1.

With above definitions the problem of optimal coordination of energy storage devices with the electric power system can be stated as follows

$$\text{Minimize} \quad x_o = \sum_{i=1}^{n} f_i(x_i, y_i, L_i, G_A) \tag{5}$$

$$\text{subject to} \quad E_o = C \tag{6}$$

$$\text{and constraint set } S_i^c, \; i = 1,2,\ldots,N \tag{7}$$

The formulated problem is rather general. It applies to any energy storage device which can be coordinated with the electric power system with single or multiple reservoirs. It also applies to the coordination of conventional hydroelectric plants with the power system. Unfortunately, the dimensionality of the model as well as the fact that certain external parameters are random variables, such as system demand L_i, externally available energy e_i, and available generators G_A, prohibit a direct solution. Fortunately, the model possesses certain properties which enable the decomposition of the problem into smaller problems, easily solvable. In the subsequent section, the decomposition technique is discussed.

Decomposition

The model possesses the following properties:

—The objective function is separable

—The constraints are also separable.

The decomposition can be effected as follows: Consider the time period T which has been divided into N intervals. Assume the period T divided into K subperiods. A subperiod is defined with the set I_k of intervals:

$$I_k = \{i \; ; \; i = n_{k+1}+1, \ldots, n_k\}$$
$$n_o = 1$$
$$k = 1,2,\ldots,K \tag{8}$$

Consider the problem

$$\text{Minimize} \quad \Lambda_\lambda = \sum_{i=1}^{n_\lambda} f_i(x_i, y_i, L_i, G_A) \tag{9}$$

$$\text{subject to} \quad E_o = c = \xi_o \tag{10}$$

$$\text{constraint set} \quad S_i^c, \; i \in I_k, \; k = 1,2,\ldots,\lambda \tag{11}$$

$$\text{and} \quad E_{n\lambda} = \xi\lambda \tag{12}$$

Let the optimal solution to above problem be

$$\Lambda_\lambda^*(\xi_o, \xi_\lambda)$$

Then the problem is formulated as a dynamic problem with the following recurrence formula

$$\Lambda_{\lambda+1}^*(\xi_o, \xi_{\lambda+1}) = \min \{\Lambda_\lambda^*(\xi_o, \xi_\lambda) + \sum_{i \in I_{\lambda+1}} f_i(x_i, y_i, L_i, G_A) \tag{13}$$

$$\text{subject to} \quad E_{n\lambda} = \xi_\lambda \tag{14}$$

$$\text{constraint set} \quad S_i^c, \; i \in I_{\lambda+1} \tag{15}$$

$$\text{and} \quad E_{n_{\lambda+1}} = \xi_{\lambda+1} \tag{16}$$

The solution to the above problem is obtained as follows: The variable ξ_λ is discretized into m values, $\xi_{\lambda i}$, i = 1, 2,..., m. For a given value $\xi_{\lambda i}$ the problem defined with (13), (14), (15), and (16) collapses into an optimization problem of the quadratic programming variety. Thus, in order to obtain the solution $\Lambda_{\lambda+1}^*(\xi_o, \xi_{\lambda+1})$, the problem defined with (13), (14), (15), and (16) is solved for every value $\xi_{\lambda i}$, i = 1, 2,..., m. The solution with the least cost will be the optimum.

In summary, application of the recurrence formula (13) requires the solution of a series of quadratic programs. The model is flexible, since the dimension of the quadratic program can be independently selected, and suitable in handling the stochastic nature of certain problem parameters. In the subsequent section a number of details of the solution method employed is outlined.

SOLUTION METHOD

For fixed ξ_λ, $\xi_{\lambda+1}$ the problem defined with (13), (14), (15), and (16) collapses to the following quadratic program

$$\text{Minimize} \quad c^t z + z^t D z \tag{17}$$

$$\text{subject to} \quad E_{n\lambda} = \xi_\lambda \tag{18}$$

$$\text{constraint set} \quad S_i^c, \; i \in I_{\lambda+1} \tag{19}$$

$$\text{and} \quad E_{n_{\lambda+1}} = \xi_{\lambda+1} \tag{20}$$

$$\text{where} \quad z = [x_{n_{\lambda+1}} \ldots x_i \ldots x_{n_{\lambda+1}} \ldots y_i \ldots y_{n_{\lambda+1}}]$$

Application of the recurrence equation (13) requires the solution of a large number of quadratic programs such as the above. Thus, the quadratic model above deserves our attention. The practicality of the method depends on how efficiently the quadratic programs can be solved. The size of the quadratic model and the fact that the constraint set S_i^c is not convex present special problems. In order to arrive to an efficient algorithm the two problems have been approached as follows.

Non-Convex Set of Constraints

The constraint set is not convex because of the constraints $\{\underline{z}_i \leq z_i \leq \bar{z}_i$ or $z_i = 0\}$. A brute force approach is to consider all possible combinations of constraint sets, compute the optimal solution and then select the best solution. Instead, the following procedure is utilized: The above constraint is replaced with $\{0 \leq z_i \leq \bar{z}_i\}$ and the resulting problem is solved. If the constraint $\{z_i = 0$ or $\underline{z}_i \leq z_i \leq \bar{z}_i\}$ is satisfied, the solution is acceptable. Otherwise, sensitivity analysis (which is readily available from the quadratic program) is employed to select constraint $\{z_i = 0\}$ or $\{\underline{z}_i \leq z_i \leq \bar{z}_i\}$ and the problem repeated. This approach has been proven to be efficient.

Size

The size of the quadratic model is greatly reduced using model reduction techniques. In this application the following procedure is followed:

(a) Omission of the term z^tDz reduces the problem defined with (17), (18), (19), and (20) into a linear program. Most of the constraints simply define bounds on the decision variables. The simplex method with upper bounds is utilized to solve the problem efficiently. Because the term z^tDz is small compared to c^tz, the obtained solution provides information regarding the effectiveness of the constraints (18), (19), and (20) in the final solution.

(b) A quadratic program is defined with the objective (17) and the effective or near effective constraints from (18), (19), and (20). (This information is known from (a)). This problem is solved with the simplex method for quadratic programming. To increase the efficiency of the solution method, constraints which impose bounds on the decision variables are implicitly handled in exactly the same way as in the simplex method with upper bounds.

The obtained solution is checked against the complete set of constraints. If violations occur, the reduced model is augmented with the violated constraints and the procedure repeated.

TEST RESULTS

The described two-level optimization model has been applied to the coordination of a 4000-MWHR reservoir pumped storage hydro station with a given electric power system. The peak load of the system during a test week is 12100 MW. The generating capability of the station is 600 MW while the pumping capability is 600 MW. Figure 2 shows a typical optimal schedule over a period of one week. Subperiods equal to the duration of one day are used in the decomposition procedure. The model provides the following additional information not shown in Figure 2.

-- Optimal water level in the reservoir
-- Electric power system fuel requirements
-- Sensitivity of economic benefits to schedule changes
-- Sensitivity of economic benefits to system parameters such as reservior capacity, generating equipment capacity, and pumping equipment capacity.

CONCLUSIONS

A decomposition technique has been described which

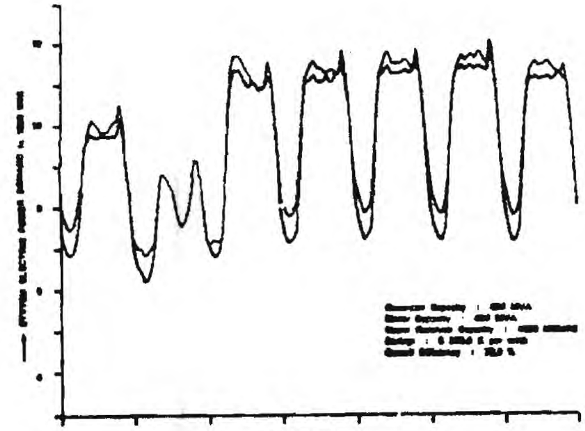


Figure 2 [illegible]

reduces the complexity of the problem of coordinating energy storage plants with an integrated electric power system. The technique results in a two level optimization procedure. At the first level quadratic programming techniques are employed to optimize plant operation over defined subperiods. At the second level dynamic programming techniques are employed to optimize plant energy storage levels at each subperiod. The described model allows exploitation of special problem structures which results in practical computational algorithms. Also, sensitivity analysis allows the optimization of plant parameters.

ACKNOWLEDGEMENTS

The work reported in this paper is supported by NSF Grant No. ENG-7816496.

REFERENCES

1. P. Wolfe, "The Simplex Method for Quadratic Programming", Econometrica, Vol. 27, 3, July 1959, pp. 382-398.

2. J. D. C. Little, "The Use of Storage Water in a Hydroelectric System", Journal of the Operations Research Society of America, Vol. III, 1955, pp. 187-197.

3. P. J. Bernard, J. F. Dopazo, and G. W. Stagg, "A Method of Economic Scheduling of a Combined Hydro and Steam Generating System", IEEE Transaction on PAS, Vol, 83, January 1964, pp. 23-30.

4. C. W. Watchorn, "Coordination of Hydro and Steam Generation", AIEE Transactions, Vol. 74, pt. III, April 1955, pp. 142-150.

5. F. A. Vizamontes and H. B. Hamilton, "Scheduling of Hydro Electric Storage Plants Using Dynamic Programming", paper presented at the IEEE PES Summer Meeting, Mexico City, Mexico, July 17-22, 1977.

6. A. P. Meliopoulos and N. Skordilis, "Coordination of Pumped Storage Stations With the Integrated Electric Power System", Proceedings of the 1979 Control of Power Systems Conference, pp. 127-131, College Station, Texas, March 19-21, 1979.

QUADRATIC PROGRAMMING WITH BOUNDED VARIABLES
AND SPARSITY TECHNIQUES

A. P. Meliopoulos Jon Vilhjalmsson

I. Abstract

The paper addresses the general quadratic programming problem with equality constraints, inequality constraints and upper bounds:

$$\text{Minimize } c^T x + x^T D x$$

$$\text{subject to} \quad A_e x = b_e$$

$$A_i x \leq b_i$$

$$x \leq h$$

Wolfe [1] has shown that his problem can be solved with a modified simplex method. This paper follows the basic Wolfe's approach but it shows that the solution can be obtained in a drastically more efficient way with the following:

a) An algorithm for the generation of a dual feasible solution

b) A modified dual simplex algorithm which incorporates implicitly the upper bounds.

The advantages of above algorithm versus Wolfe's algorithm are documented.

Application of sparcity techniques to the described algorithms is discussed. It is shown that the developed algorithm is extremely suitable to sparcity techniques resulting in highly efficient quadratic program solution times.

The algorithm has been developed for the problem of stochastic co-ordination of energy storage devices with an integrated power system [7]. This problem leads to stochastic recurrence formulae of the quadratic programming variety. The solution of these formulae require the analysis of

numerous quadratic programs and performance of sensitivity analysis. The developed algorithms have been successfully applied to this problem.

2. Introduction

A number of problems of interest can be solved in a quadratic programming formulation framework. Some of these are:

> Regression. To find the best least-squares fit to given data, where certain parameters are known a priori to satisfy inequality constraints (e.g., being nonnegative).
>
> Efficient production. Maximization of profit, assuming linear production functions and linearly varying marginal costs.
>
> "Portefolio" problem. To find a combination of random variables having given expectation and minimum variance.
>
> Convex programming. To find the minimum of a general convex function under linear constraints using a quadratic approximation.

A few applications can be mentioned within the class of efficient production problems. Examples from the power systems area are:

a) Economic Dispatch. The economic dispatch problem can be formulated as a quadratic program if quadratic production cost functions are assumed for the generating units as done by Reid and Hasdor [3].

b) Hydro-Thermal Coordination. If again quadratic production cost functions of thermal generating units are assumed then the hydro-thermal coordination problem can be solved by quadratic programming [7].

c) Supply Management. Energy storage stations, such as pumped hydrostorage stations, can be used to modify the system load as seen by the generation system. The production cost functions of the generation system can be assumed quadratic and the problem can be formulated as a quadratic programming problem.

d) Pollution Dispatch. Pollution dispatch can be formulated in a similar manner as the economic dispatch problem where a quadratic factor is added to the production cost function to account for the pollution [6].

Wolfe [1] has shown that the quadratic programming problem can be transformed into solving a modified linear programming problem in which case the simplex method of linear programming can be used. This approach leads to a dimensionally large linear program. The efficiency of the method can be drastically increased with the introduction of a dual simplex algorithm and sparcity techniques. The resulting method is especially attractive to problems requiring the solution to numerous slightly perturbed quadratic programs. Such situations are common in stochastic efficient production problems.

This paper describes a general algorithm for solving quadratic programming problems with bounded variables and the associated application of sparcity techniques.

3. Formulation

Consider the general quadratic program described with objective (1) and constraints (2), (3), (4) and (5).

$$\text{Minimize } f(x) = c^T x + x^T Dx \tag{1}$$

$$\text{subject to:} \quad A_i x \leq b_i \tag{2}$$

$$A_e x = b_e \tag{3}$$

$$x \leq h \tag{4}$$

$$x \geq 0 \tag{5}$$

where x, c and h are vectors of dimension n

D is an (nxn) matrix

b_i and b_e are vectors of dimension n_i and n_e respectively

A_i and A_e are matrixes of dimension $n_i$xn and $n_e$xn respectively.

The Kuhn-Tucker conditions provide a necessary condition for a minimum [2]. By applying the Kuhn-Tucker conditions the problem is transformed into finding a feasible solution to the following set of equations and constraints:

$$C + 2Dx + A_i^T \mu^{(1)} + A_e^T \lambda + I\mu^{(2)} - I\nu = 0 \tag{6}$$

$$A_i X + S^{(1)} = b_i \tag{7}$$

$$A_e X = b_e \tag{8}$$

$$\mu^{(2)T}(X-h) = \mu^{(2)T} S^{(2)} = 0 \tag{9}$$

$$\mu^{(1)T}(A_i X - b_i) = \mu^{(1)T} S^{(1)} = 0 \tag{10}$$

$$\nu^T X = 0 \tag{11}$$

$$X, \nu, \mu^{(1)}, \mu^{(2)}, S^{(1)}, S^{(2)} \geq 0 \tag{12}$$

$$\lambda \text{ free} \tag{13}$$

where

$\mu^{(1)}$: n_i x 1 vector

$\mu^{(2)}$: n x 1 vector

ν : n x 1 vector

λ : n_e x 1 vector

$S^{(1)}$: n_i x 1 vector

$S^{(2)}$: n x 1 vector

In a matrix form this becomes

$$\begin{bmatrix} -2D, & -A_i^T, & -A_e^T, & -I, & I, & 0, & 0 \\ A_i, & 0, & 0, & 0, & 0, & I, & 0 \\ A_e, & 0, & 0, & 0, & 0, & 0, & 0 \\ I, & 0, & 0, & 0, & 0, & 0, & +I \end{bmatrix} \begin{bmatrix} x \\ \mu^{(1)} \\ \lambda \\ \mu^{(2)} \\ \nu \\ S^{(1)} \\ S^{(2)} \end{bmatrix} = \begin{bmatrix} C \\ b_i \\ b_e \\ h \end{bmatrix} \tag{14}$$

$$\nu^T X = 0 \tag{15}$$

$$\mu^{(1)T} S^{(1)} = 0 \tag{16}$$

$$\mu^{(2)T} S^{(2)} = 0 \tag{17}$$

Where the dimension of the matrix is $(2n + n_i + n_e) \times (4n + 2n_i + n_e)$. The following should be noted:

a) The free variable λ can be substituted with $\lambda = \lambda^+ - \lambda^{-1}$ where $\lambda^+, \lambda^- \geq 0$ and λ^+, λ^- are n_e x 1 vectors.

b) The equation $X + S^{(2)} = h$ imposes upper bounds on the variables X. This constraint can be implicitly taken into consideration with the introduction of the following transformation

$$X_i^+ = X_i$$

$$X_i^- = h_i - X_i$$

and as the method progresses X_i^+ or X_i is used depending on whether the variable X_i has most recently been at its lower or upper bound, respectively. (Simplex method with upper bounds.)

With the above observations the problem is reduced to the following

$$\begin{bmatrix} -2D, & -A_i^T, & -A_e^T, & A_e^T, & -I, & I, & 0 \\ A_i, & 0, & 0, & 0, & 0, & 0, & I \\ A_e, & 0, & 0, & 0, & 0, & 0, & 0 \end{bmatrix} \begin{bmatrix} X \\ \mu^{(1)} \\ \lambda^+ \\ \lambda^- \\ \mu^{(2)} \\ \nu \\ s^{(1)} \end{bmatrix} = \begin{bmatrix} C \\ b_i \\ b_e \end{bmatrix} \qquad (18)$$

$$\nu^T X = 0 \qquad (19)$$

$$\mu^{(1)T}\, s^{(1)} = 0 \qquad (20)$$

$$\mu^{(2)T}\, (h - x) = 0 \qquad (21)$$

$$X,\ \mu^1,\ \mu^2,\ \lambda^+,\ \lambda^-,\ \nu\ ,\ s^1 \geq 0$$

and upper bounds on the variables X:

$$X \leq h \qquad (22)$$

Where the dimension of the matrix is now $(n + n_i + n_e) \times (3n + 2n_i + 2n_e)$.

The following notation can now be introduced:

$$A = \begin{bmatrix} -2D, & -A_i^T, & -A_e^T, & A_e^T, & -I, & I, & 0 \\ A_i, & 0, & 0, & 0, & 0, & 0, & I \\ A_e, & 0, & 0, & 0, & 0, & 0, & 0 \end{bmatrix}$$

$$y = \begin{bmatrix} x \\ \mu^{(1)} \\ \lambda^{+} \\ \lambda^{-} \\ \mu^{(2)} \\ \nu \\ s^{(1)} \end{bmatrix} \qquad b = \begin{bmatrix} c \\ b_i \\ b_e \end{bmatrix}$$

$$n_b = n + n_i + n_e$$
$$n_x = 3n + 2n_i + 2n_e$$

where A: $n_b \times n_x$ matrix

y: $n_x \times 1$ vector

b: $n_b \times 1$ vector

The matrix A can be represented by column vectors a^i as follows

$$A = [a^{(1)}, a^{(2)}, \ldots, a^{(n_x)}]$$

where all the vectors a^i, $i = 1, \ldots, n_x$ are of dimension n_b.

A feasible solution to the equations (17), (18), (19), (20) and (21) will be the optimal solution to the general quadratic problem defined with (1), (2), (3), (4), and (5). In subsequent paragraphs two algorithms will be described for the computation of the solution. The simplex procedure and the dual simplex procedure are the bases of these algorithms. Implementation of these algorithms will be discussed and will be shown that the algorithm based on the dual simplex procedure has significant advantages when it is implemented with sparcity techniques.

4. Solution Algorithms

4.1 Solution Algorithm Based on the Primal Simplex Procedure

If n_b artificial variables r are introduced to construct a starting feasible solution then the problem can be formulated as a linear program with objective function equal to the sum of artificial variables and with exclusivity constraints (18), (19) and (20). This results in the following linear program:

$$\text{Minimize} \sum_{j=1}^{n_b} r_j = [\ C\ ]^T \begin{bmatrix} y \\ r \end{bmatrix} \tag{23}$$

$$\text{subject to } AY + r = b \tag{24}$$

$$Y_i,\ r_j \geq 0 \quad i = 1, 2, \ldots, n_x,\ j = 1, 2, \ldots, n_b \tag{25}$$

and exclusivity constraints

$$\nu_i x_i = y_i y_{i+\ell_1} = 0 \qquad i = 1, 2, \ldots, n \tag{26}$$

$$\ell_1 = 2n_b - n_i$$

$$\mu^{(1)}_{i-N}\, s^{(1)}_{i-N} = y_i y_{i+\ell_1} = 0 \qquad i = n + 1, \ldots, n + n_i \tag{27}$$

$$\mu_i^{(2)} (h_i - x_i) = y_{i+\ell_2} (h_i - y_i) = 0 \tag{28}$$

$$\ell = 1, 2, \ldots, n$$

$$\ell_2 = n_b + n_e$$

where C : $(n_x + n_b) \times 1$ vector

r : $(n_b \times 1$ vector

The following algorithm results from the application of the primal simplex procedure appropriately modified to this problem:

1) A starting extended feasible solution is given by

$r = b$

$y = 0$

2) Select a variable, which if enters the solution will improve the objective function, and which will not violate the exclusivity constraints. Thus a variable k in order to qualify for entering variable has to pass the following two tests.

2a) Constraints (25), (26), and (27) are not violated

2b) The reduced cost coefficient is less than zero

$$r_k = -C_B^T \quad B^{-1} \; a^k \; e_k \leq 0 \tag{29}$$

where

C_B : is the vector of the objective function coefficients of the basic variables. The i^{th} entry is one if the i^{th} basic variable is an artificial variable, otherwise zero.

B : is the matrix of the basis. The columns of B are the columns of the matrix A that correspond to the basic variables.

a^k : is the vector of the matrix A corresponding to variable k.

$$e_k \begin{cases} +1 \text{ if the variable k is at its lower bound} \\ -1 \text{ if variable k is at its upper bound.} \end{cases}$$

If no variable can be found that satisfies both tests then the algorithm terminates. If all the artificial variables r are zero then a solution has been found. Proof for above termination condition can be found in [1].

3) Determine the variable which can leave the basis without forcing the new solution to become feasible. To this purpose compute

$$S = B^{-1} b \tag{30}$$

$$\alpha = B^{-1} a^k \tag{31}$$

$$\varepsilon_1 = h_x \tag{32}$$

$$\varepsilon_2 = \min_i \{\frac{S_i}{\alpha i}, \alpha_i > 0\} \quad , \quad \text{index } j \tag{33}$$

$$\varepsilon_3 = \min_i \{\frac{S_i - h_i}{\alpha i} \quad , \quad \alpha_i < 0\} \quad , \text{index } j \tag{34}$$

If $\varepsilon_1 \leq \varepsilon_2$ and $\varepsilon_1 \leq \varepsilon_3$ go to 4

If $\varepsilon_2 < \varepsilon_1$ and $\varepsilon_2 \leq \varepsilon_3$ go to 5

If $\varepsilon_3 < \varepsilon_1$ and $\varepsilon_3 < \varepsilon_2$ go to 6

4) The variable k goes to its opposite bound. Basis remains the same. Change sign of variable e_n

$$e_n = -e_n \; ,$$

and update the b vector.

$$b = b + h_k a^k e_n \tag{35}$$

Go to step 2.

5) The j^{th} basic variable becomes nonbasic and returns to its old bound. The k^{th} variable enters the basis. Replace the vector a^j in the B matrix with the vector a^k. Go to step 2.

6) The j^{th} basic variable becomes nonbasic and goes to its opposite bound. The k^{th} variable enters the basis. Change sign of variable e_j.

$$e_j = -e_j$$

Update vactor b

$$b = b + h_j \, a^j \, e_j \tag{36}$$

Replace the vector a^j in the B matrix with the vector a^k. Go to step 2.

Above algorithm is straightforward. It can be implemented with sparcity techniques which speed up the computations. It has the disadvantage, however, of requiring approximately $2n_b$ iterations to terminate. Sparcity techniques drastically decrease the computational effort per iteration, but they have no effect on the number of iterations required. Thus the efficiency of the solution method will be drastically increased if the number of iterations is minimized. This possibility is exploited with the algorithm described in the next section.

4.2 Solution Algorithm Based on the Dual Simplex Procedure

The dual simplex procedure can be employed for the solution of the problem defined with (17), (18), (19), (20) and (21). The algorithm will be developed by assuming that a dual feasible solution satisfying the exclusivity constraints is known. Then a procedure to secure such a solution will be outlined. In particular this solution has the following properties.

$$y_i y_{i+\ell_i} = 0 \qquad i = 1, 2, \ldots, \qquad \ldots, n+n_i$$

$$y_{i+\ell_2}(h_i - y_i) = 0 \qquad i = 1, 2, \ldots, n$$

$$\ell_1 = 2n_b - n_i$$

$$\ell_2 = n_b + n_e$$

The algorithm can be summarized as follows:

Step 1: Scan solution y to find the maximum violation of the following constraints

1a) $y \geq 0$

1b) $y \leq h$

If constraints (1a) and (1b) are satisfied stop. Solution has been found. If constraints (1a) are violated let j be the most negative variable and go to step 2. If constraints (1b) are violated let j be the variable with the maximum $y_j - h_j$ value, and go to step 3.

Step 2: Scan all the nonbasic variables, i, excluding the ones which will violate the constraints

$$y_i y_{i+\ell_i} = 0 \quad i = 1, 2, \ldots, n+n_i$$

$$y_{i+\ell_2}(h_i - y_i) = 0 \quad i = 1, 2, \ldots, n$$

if assume a nonzero value, computing for each one the following quantity:

$$\varepsilon_k = \max_i \{y_{jk} \quad , y_{ji} < 0\}$$

where

$$y^i = B^{-1} a^i$$

Let k be the index i minimizing the above number. Then compute

$$\varepsilon_1 = h_k$$

$$\varepsilon_2 = \frac{S_j}{y_{jk}}$$

where S_j is the j^{th} entry of the basic solution vector $B^{-1}b$

Then

a) if $\varepsilon_1 \leq \varepsilon_2$ variable k goes to opposite bound. Basis remains unchanged.

b) if $\varepsilon_1 > \varepsilon_2$ the j^{th} basic variable leaves the basis. Variable k enters the basis.

Step 3: Scan all the nonbasic variables, i, excluding the ones which will violate the contraints

$$y_i y_{i+\ell_1} = 0 \quad i = 1, 2, \ldots, n+n_1$$

$$y_{i+\ell_2}(h - y_i) = 0 \quad i = 1, 2, \ldots, n$$

if assume a nonzero value, computing for each one the following quantity:

$$\varepsilon_k = \min_i \{y_{ji} \quad , y_{jk} > 0\}$$

where

$$y^i = B^{-1} a^i$$

Let k be the index i minimizing the above ratio. Then compute

$$\varepsilon_1 = h_k$$

$$\varepsilon_2' = (S_j - h_j)/y_{jk}$$

where again S_j is the j^{th} entry of the basic solution $B^{-1}b$. Then

a) if $\varepsilon_1 \leq \varepsilon_2$ variable k goes to opposite bound. Basis remains unchanged.

b) if $\varepsilon_1 > \varepsilon_2$ the j^{th} basic variable goes to opposite bound. Variable k enters the basis.

The above algorithm has been derived as follows: The problem of finding a solution to the set of equations

$$Ay = b$$

subject to

$$y_i y_{i+\ell_1} = 0 \qquad i = 1, 2, \ldots, n+n_1$$

$$y_{i+\ell_2}(h_i - y_i) = 0 \qquad i = 1, 2, \ldots, n$$

$$y \leq h$$

is formulated as the following linear program

Minimize $C^T y$

s.t. $Ay = b$

$$y_i y_{i+\ell_1} = 0 \qquad i = 1, 2, \ldots, n+n_1$$

$$y_{i+\ell_2}(h_i - y_i) = 0 \qquad i = 1, 2, \ldots, n$$

$$y \leq h$$

The choice of the constant vector C is arbitrary. If however the vector C is selected in such a way that all the reduced cost coefficients for the above problem and for the current basic solution are positive and equal and the the dual simplex algorithm is applied, the previously described algorithm results. This trick is employed to circumvent the fact that the dual problem to the above linear program is degenerate.

The advantage of the above algorithm is in the fact it can start from any basic solution as long as the exclusivity constraints are satisfied. Such a solution can be formed immediately. If in addition this solution in close to the optimal, then few iterations will yield the solution. Many times a basic starting solution can be found which is close to the optimal (see parametric solutions).

5. Existence of Solution

The existence of a solution to the problem defined by equations (1) - (5) can be considered in two parts.

a) The matrix D is positive semidefinite. In this case there always exists a solution if the solution space defined by equations (2) - (5) is not empty. The solution may be bounded or unbounded. If in addition the matrix D is positive definite the solution will be bounded.

b) The matrix D is not sign definite and the space of the constraints is not empty and bounded. Then there always exists a solution.

6. Implementation

Because matrix A in equation (17) is highly sparse, the described algorithms have been sparsity coded. Sparsity techniques are efficient in forming matrices and vectors required in the application of the algorithm and thus drastically increasing computational efficiency. In this paragraph, some implementation details will be discussed briefly and then the advantages of this procedure will be discussed.

The heart of the sparsity coded quadratic program is the solution of the following basic equation

$$Bx = b$$

where B is the matrix of the basis, b is a driving vector and x the solution. The matrix of the basis is stored in sparse form with the diagonal and off-diagonal elements of the matrix respectively. A general sparsity coded LU factorization procedure factors the matrix B into the product of a lower and an upper diagonal matrix. Forward and back substitution with the elements of these matrices provide the solution vector x in an extremely efficient manner. A number of supporting sparsity coded software can modify the matrix of the basis by simply replacing appropriate entries of the matrix. In addition a number of vectors which completely define the solution to the quadratic problem at each iteration are utilized. These are:

IBASE : Pointer of basic variables. If the i^{th} element of this vector is k, then the k^{th} variable is basic and column i of B corresponds to column k of A.

JBASE : A code defining if a variable is basic or non-basic

= 1, variable is basie

= 0, variable is nonbasic

IBND : A code defining if variable is at its lower or upper bounds.

= 1, variable is the original variable

= -1, variable has been transformed with

$$X^{+} = -X^{-} + h$$

ILC1 : A code defining if variable is zero or nonzero.

= 1, variable is nonzero

= 0, variable is zero.

ILC2 : A code defining if a variable qualifies for entering the basis.

= 1, variable qualifies for entering

= 0, variable does not qualify for entering.

The vectors ILC1 and ILC2 are used for enforcing the exclusivity constraints. In each iteration all these vectors are updated. With the defined vectors and supporting software, the implementation of the primal and dual algorithms is very simple. In the beginning the previously mentioned vectors and the matrix B are initialized. Initialization of matrix B requires that matrix A is stored and available (also in sparse form). Then at every iteration the mentioned vectors and matrix B are simply updated according to the algorithm, until the termination conditions have been met. At every iteration one LU factorization and several forward and back substitutions are required. These computations are extremely efficient in sparse form.

7. Degeneracy

Degeneracy is detected whenever a basic variable meets the following condition

$$|X_i| < \varepsilon$$

or

$$|X_i - h_i| < \varepsilon$$

where ε is a very small number.

In this case the degeneracy is alleviated by setting

$$X_i = 2\varepsilon$$

or

$$X_i = h_i - 2\varepsilon \quad \text{respectively}$$

and updating the driving vector b as follows

$$b \rightarrow b + 2\varepsilon a^i$$

where a^i is the column of matrix B corresponding to the i^{th} basic variable.

8. Parametric Solutions

Many times there is a need to solve quadratic programs which result from small perturbation of the parameters involved. Efficiency is of paramount inportance in this case. The problem can be addressed in a number of ways:

a) Sensitivity analysis can be employed to project the new solution due to parameter changes. This approach is accurate only if the parameter changes do not cause changes in the solution basis.

b) The modified dual simplex procedure (second algorithm) provides an efficient procedure for computing exact solution to perturbed problems. To this purpose the matrices A and B are updated with the new parameter values (by updating a number of appropriate entries). Then the new basic solution is computed. If the solution is also feasible, it represents the new optimal basic feasible solution. Otherwise the second algorithm (dual simplex) is applied. Depending on the size of the perturbation the optimal solution will be found in few iterations.

The approach (b) has been applied to the problem of stochastic optimal coordination of energy storage devices with electric power systems. A dramatic increase in efficiency has been recorded. Execution time was decreased by a factor of 100 as compared to complete solutions [7].

For the sake of completeness Wolfe [1] has given a parametric solution to the quadratic programming problem (long form). However this analysis involves only one parameter and thus limited. The algorithm of this paper based on the dual simplex procedure is a generalization of the concept of parametric analysis.

9. Conclusions

This paper presents two solution algorithms to the general quadratic programming problem. One is based on the primal simplex procedure and the other on the dual simplex procedure. The problem of computational efficiency is addressed. Efficiency is drastically increased in two ways: a) Implicit incorporation of bounds on the variables and, b) sparsity techniques. Finally the developed algorithms provide efficient procedures for generalized parametric analysis of quadratic programs.

10. References

[1]. P. Wolfe, "The Simplex Method for Quadratic Programming", Econometrica, vol. 27, 3, pp 382-398, July, 1959.

[2]. D. G. Luenberger, "Introduction to Linear and Nonlinear Programming", Addison-Wesley Publishing Corporation, 1973.

[3]. G. F. Reid and L. Hasdorff, "Economic Dispatch Using Quadratic Programming", Paper presented at the IEEE-PES Winter Meeting, New York, NY, January 28-February 2, 1973.

[4]. A. P. Meliopoulos Progress Report, National Science Foundation Grant No. ENG-7816496, "Optimal Coordination of Energy Storage Facilities with an Integrated Electric Power System".

[5]. E. W. Barankin, R. Dorfman, "On Quadratic Programming", University of California Publications in Statistics, 2 (1958), 285-318.

[6]. O. E. Finnigan and A. A. Fouad, "Economic Dispatch with Pollution Constraints", Paper presented at the IEEE-PES Winter Meeting, New York, NY, January 27-February 1, 1974.

[7]. A. P. Meliopoulos, J. Vilhjalmsson, "Optimal Coordinating Policies of Pumped Hydrostorage Plants in the Presence of Uncertainty", Paper submitted for publication in the IEEE Transactions on Power Apparatus and Systems.

OPTIMAL COORDINATING POLICIES OF PUMPED HYDROSTORAGE PLANTS IN THE PRESENCE OF UNCERTAINTY

A. P. Meliopoulos
Member IEEE

Jon Vilhjalmsson
Student Member, IEEE

School of Electrical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

1. ABSTRACT

This paper describes a two level stochastic optimization procedure for the computation of optimal coordination strategies of pumped hydrostorage plants with integrated electric power systems. At the first level a quadratic programming procedure optimizes the operation of pumped hydrostorage plants over a period of one day for specified terminal stored energy levels. At the second level a dynamic programming successive approximations procedure determines optimal stored energy levels at the beginning of each day. Uncertainty resulting from generating unit forced outages, electric load and run of the river variations is incorporated at this level. The method has been implemented using sparcity techniques. Results obtained with a realistic 46-generating unit, 11,900 MW peak load system are included. The results indicate that the impact of generating unit forced outages and load uncertainty on storage plant scheduling is substantial. In general, thermal unit outages amplify the economic benefits from storage plants. Also for a given electric power system there exists an optimal total capacity of storage plants. The methodology of this paper is applicable for scheduling other storage plants and can be used for operations planning as well as expansion planning.

2. INTRODUCTION

The general object of coordination of pumped hydrostorage plants with electric power systems is the minimization of the overall production cost and the maximization of generation reserves. The dominant feature of pumped hydrostorage plants (or any other storage plants) is the capability of replacing electric energy produced from high grade fuel with electric energy produced from more abundant and therefore less expensive fuels. Pumped hydrostorage stations provide an economically feasible solution to electric supply management.

Once a pumped hydrostorage station is installed, its economic operation as an integral component of an electric power system requires the selection of a pumping and generating schedule which will result in minimum overall production cost. This selection must recognize the following:

(a) the uncertainty of availability and loading capability of the generating units during a given period, especially of the base load units;

(b) the incremental production cost characteristics of these units;

(c) the expected hourly system demand and associated uncertainty;

(d) interconnection transactions;

(e) projected river inflow in case the station is on the run of a river;

(f) effect of transmission losses; and

(g) effect of hydraulic head variations.

This is known as the coordination problem. The problem has been extensively studied and several models [6], [7], [8], [11], [12], [13] have been developed. In general these models are complex, impose heavy computational requirements and ignore the uncertainty associated with the parameters of the problem such as generating unit availability and load variations. In particular, reference [13], presents a deterministic linear programming formulation of the problem of energy storage device scheduling. This approach is realistic only for small capacity plants. For large capacity pumped hydrostorage plants, the energy replacement cost has a significant quadratic component. Linear programming formulations ignore this component leading to overestimation of economic benefits resulting from storage plants. Viramontes and Hamilton [12] have applied dynamic programming techniques to the scheduling problem of storage plants. They treated the deterministic problem only and they did not address the problem of computational efficiency, which is extremely important in all dynamic programming approaches.

This paper presents a new formulation of the problem and a decomposition technique leading to a two level optimization procedure. At the first level a quadratic programming formulation is employed to define the optimal coordination over a period of one day assuming known the stored energy in the upper reservoir at the beginning and end of the day. At the second level a dynamic programming formulation is employed to determine target stored energy levels in the upper reservoir at the beginning of each day. At this level uncertainty in load levels and unit availability can be easily incorporated. This formulation and decomposition results in computationally efficient and practical algorithms.

In subsequent sections the formulation is defined, and the two level optimization procedure is described. Finally, results with a realistic test system are presented and discussed. In particular, the impact of thermal units forced outages is emphasized.

3. PROBLEM DESCRIPTION

Coordination between pumped hydrostorage plants and an electric power system means the minimization of the overall operating cost of the system. To be determined is the policy under which these plants should operate such that the production cost of the thermal generating units is minimized. Because the operation of an integrated electric power system is restrained with numerous constraints the stated coordination problem is a complex and dimensionally large optimization problem. In addition, the optimal coordination policy for a pumped hydrostorage plant depends on thermal unit availability and demand for electric power. The uncertainty associated with these two parameters makes the problem a stochastic optimization problem.

To reduce complexity and size of the stated problem, a decomposition technique [1] has been introduced which partitions the coordination problem into two manageable subproblems:

Subproblem 1

It is defined as the optimal scheduling of pumped hydrostorage stations over a time period of one day (i.e. from 8:00 a.m. to 8:00 a.m. of next day). For this subproblem it is assumed:

(1) the stored water level in the upper reservoir at the beginning and the end of the day is specified, V_1, and V_2 respectively;
(2) the available thermal generating units and their cost curves are known. This information is symbolically denoted with the set of available generating unit data G_A;
(3) the total projected system load and its variation for the duration of the day is known; and
(4) other related information, for example power imports, water flown into the upper reservoir from rivers, etc., is assumed to be known for the duration of the day.

The stated subproblem 1 is formulated as a quadratic programming optimization problem in the next section. This subproblem is utilized in subproblem 2.

Subproblem 2

A second level optimization model is developed which utilizes the mathematical model of subproblem 1 in order to determine optimal water levels in the upper reservoir at the beginning of each day. This subproblem employs a dynamic programming solution technique. Uncertainty of various parameters impacting on the optimal operation of pumped hydrostorage plants is incorporated. Specifically, thermal unit forced outages and electric load uncertainty is explicitly modeled.

In the next section the complete formulation of the two subproblems is described.

4. FORMULATION

Subproblem 1

The problem of optimal coordination of energy storage devices, over a time period of one day with specified water level in the upper reservoir at the beginning and the end of the day is considered. Constraints of energy availability, hydraulic head variations, equipment capacities and electric power demand are imposed. The point of departure in formulating the problem is the chronological system load curve which is discretized as in Figure 1. The time period is divided into N intervals of short duration (for practical reasons intervals of one hour are selected). It is assumed that the electric power demand remains constant for the duration of one interval. During a given interval, a pumped hydrostorage plant may operate in the pumping or generation mode absorbing or delivering x or y MW of power respectively as illustrated in Figure 2. Thus two cases need to be considered:

(a) Pumping Mode. Assume interval i with average electric load L_i and that the hydrostorage plant operates so that absorbs x_i MW from the electric power system as it is shown in Figure 2. The cost of operating the plant will be

$$C_{pi} = \tau_i \int_{z=0}^{x_i/\eta_{TR}} g(L_i' + z)dz$$

where

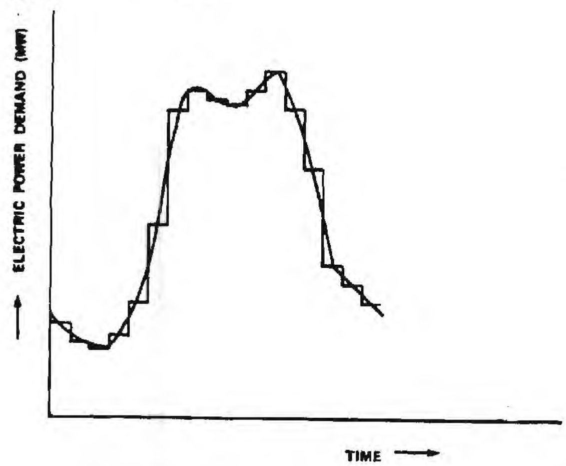


Figure 1 Discretization of the Chronological Electric Load Curve

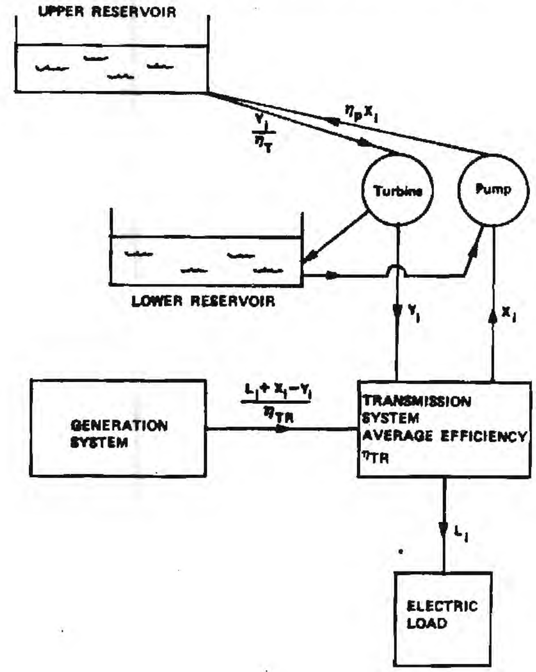


FIGURE 2. A Snapshot of the Operation of a Pumped Hydrostorage Plant.

η_{TR} average transmission system efficiency

$g(L_i' + z)$ the incremental production cost of electric power at total generation level $L_i' + z$, where $L_i' = L_i/\eta_{TR}$

τ_i is the duration of interval i.

Because $L_i' >> x_i$, the incremental production cost function g can be appoximated with

$$g(L_i' + z) \doteq g(L_i') + \frac{dg(L_i')}{dL_i} z$$

Back substitution into the integral will yield

$$C_{pi} = \tau_i g(L_i') \frac{x_i}{\eta_{TR}} + \tau_i \frac{dg(L_i')}{dL_i'} \frac{x_i^2}{2\eta_{TR}^2}$$

$$= a(L_i')x_i + b(L_i')x_i^2$$

Above expression represents the cost of operating the plant in the pumping mode at level x_i during interval i. It should be observed that this cost is a quadratic function of the operating level x_i. For small capacity plants the quadratic component $b(L_i')x_i^2$ is negligible with respect to the linear component $a(L_i')x_i$. However, for the capacities of existing pumped hydrostorage plants (above 200 MW) the quadratic component is substantial. Omission of this component, as in [13], leads to inaccurate prediction of production costs.

(b) Generation Mode. Similarly, as in (a), assuming interval j, with average electric load L_j and that a hydrostorage plant is in generation mode generating y_j MW the cost of the replaced energy is computed to be

$$C_{gj} = -a(L_j')y_j + b(L_j')y_j^2$$

Now, let I be the set of intervals at which the hydrostorage plant operates in the pumping mode and J be the set for generation mode. The restitution cost, R_c, defined as the cost of energy stored/retrieved during the pumping/generation cycle will be:

$$R_c = \sum_{i\epsilon I} c_{pi} + \sum_{j\epsilon J} c_{gj}$$

In addition, define the following variables:

$\bar{x}$ capacity of pumping equipment

$\underline{x}$ minimum allowable level of operation of pumping equipment

E_{i-1} water level in the upper reservoir at the end of interval i-1

η_p average efficiency of pump/motor system including hydraulic losses

E_{max} capacity of the upper reservoir

E_{min} minimum allowable stored energy level in the upper reservoir

e_i water volume available from rivers in the i-th interval

η_T average efficiency of turbine/generator system including hydraulic losses

$\bar{y}$ capacity of the turbine/generator system.

$\underline{y}$ minimum allowable level of operation of turbine/generator system.

where n is the number of intervals in one day.

With above definitions the problem of optimal coordination of hydrostorage plants with electric power systems can be stated as follows:

$$\text{Minimize } J = \sum_{i\epsilon I} \{a(L_i')x_i + b(L_i')x_i^2\} + \sum_{j\epsilon J} \{-a(L_j')y_j + b(L_j')y_j^2\} \quad (1)$$

subject to:

$$\left.\begin{array}{l} \underline{x} \le x_i \le \bar{x} \quad \text{or} \quad x_i = 0 \\ E_{i-1} + \eta_p x_i \tau_i + e_i \le E_{max} \end{array}\right\} \quad (2)$$

$$\left.\begin{array}{l} \underline{y} \le y_j \le \bar{y} \quad \text{or} \quad y_j = 0 \\ E_{min} \le E_{j-1} - \frac{y_j \tau_i}{\eta_T} + e_j \le E_{max} \end{array}\right\} \quad (3)$$

$$E_o = V_1 \quad (4)$$

$$E_n = V_2 \quad (5)$$

In above formulation the objective is defined as the minimization of the restitution cost. Constraints (2) observe the operating limits of the pump/motor system and that the stored energy in the upper reservoir will not exceed the capacity of the reservoir. Similarly, constraints (3) observe the operating limits of the turbine/generator system and the capacity limits of the upper reservoir. Constraints (4) and (5) impose the assumed stored energy level at the beginning and end of the day under consideration.

The defined optimization problem is of the quadratic programming variety (quadratic objective, linear constraints). An outline of the solution method employed is given in the next section after the description of subproblem 2.

Subproblem 2

The object of this subproblem is the determination of target energy storage levels at specified time points for example at the beginning of each day. Problem parameter uncertainty, namely load level uncertainty, generating unit availability and pumped hydrostorage plant availability, are incorporated.

A stochastic optimization procedure is readily developed with the aid of dynamic programming and the described model for subproblem 1. In particular let

$$\Lambda^*(V_k, V_{k+1}, G_A, L)$$

represent the optimal cost of the solution of the problem defined with (1), (2), (3), (4), and (5) for day k. G_A is the set of available for production thermal units and L represents the assumed hourly load levels for day k. The second level optimization procedure is defined with

$$\text{Minimize } J = E \sum_k \Lambda^*(V_k, V_{k+1}, G_A, L)$$

subject to $E_{min} \leq V_k \leq E_{max}$, $k=1,2,\ldots$

The index k scans the days in the planning period. The expectation is with respect to unit availability, G_A, and load distribution, L. A Markov model is assumed for unit availability which is described in Appendix II and an autoregressive model for the load which is described in Appendix I.

The dynamic programming formulation leads to the following forward recurrence formula

$$J^*_{\lambda+1}(V_{\lambda+1}) = \min_{V_\lambda}[J^*_\lambda(V_\lambda) + E\{\Lambda^*(V_\lambda, V_{\lambda+1}, G_A, L)\}] \quad (6)$$

$$E_{min} \leq V_\lambda \leq E_{max} \quad (7)$$

where $J_\lambda^*(V_\lambda)$ is the optimal cost of operation up to day λ. The above recurrence formula can be easily solved by utilizing the first subproblem since it provides an efficient procedure for the computation of the quantity $E[\Lambda^*(V_\lambda, V_{\lambda+1}, G_A, L)]$.

Formulation for m Storage Plants

The presented formulation can be extended to electric power systems with more than one pumped hydrostorage plants. Examine for example subproblem 1. Assume there are m pumped hydrostorage plants. It will be sufficient if in the objective function (1) the variables x_i, y_j, be replaced with $\sum_{\ell=1} x_i(\ell)$, $\sum_{\ell=1} y_j(\ell)$ respectively. Where $x_i(\ell)$, $y_j(\ell)$ are the level of operation (pumping, generation respectively) of the ℓ-th storage plant. In addition, for each plant a set of constraints should be written similar to (2) and (3). If two or more plants are on the same run of a river appropriate linear constraints are introduced.

Subproblem 2 is also easily extended to the multistorage plant case. The resulting recurrence formula is:

$$J^*_{\lambda+1}(V^{(1)}_{\lambda+1},\ldots,V^{(m)}_{\lambda+1}) = \min_{V^{(1)}_\lambda,\ldots,V^{(m)}_\lambda}[J^*_\lambda(V^{(1)}_\lambda,\ldots,V^{(m)}_\lambda) +$$

$$+ E\{\Lambda^*(V^{(1)}_\lambda, V^{(1)}_{\lambda+1},\ldots,V^{(m)}_\lambda, V^{(m)}_{\lambda+1}, G_A, L)\}] \quad (8)$$

$$\begin{gathered} E^{(1)}_{min} \leq V^{(1)}_\lambda \leq E^{(1)}_{max} \\ \cdots\cdots\cdots \\ E^{(m)}_{min} \leq V^{(m)}_\lambda \leq E^{(m)}_{max} \end{gathered} \quad (9)$$

where the superscript refers to the pumped hydrostorage plant.

5. TREATMENT OF STOCHASTIC VARIABLES

The defined model for optimal coordination of hydrostorage plants with electric power systems involves a number of stochastic variables namely the electric load, the availability of generating units, river inflow, and the availability of the storage plant itself. Load uncertainty is accounted for with an autoregressive model of the hourly load level which is described in Appendix I. Thermal generating unit forced outages are accounted by adopting a truncated Markov model for the generating system. This model results in a number of discrete incremental production cost functions similar to those of Figure 4. At a given time in future the probability of existence of these curves can be computed from the Markov model. The model is presented in Appendix II.

Depending on the computation of the expected value of Λ^* in the recurrence formula (6) or (8), alternative policies for the operation of storage plants can be defined. These policies have been studied by Henault [15] for the mathematically similar problem of transmission expansion planning. A number of computational approaches have been proposed by Henault. In this paper two of the alternate approaches described in [15] are considered. In addition, the deterministic case is also cited for completeness:

(a) The deterministic approach is which Λ^* is computed assuming deterministically projected hourly electric load and a generating system without forces outages;

(b) The certainty equivalent approach is which Λ^* is computed assuming an hourly electric load equal to its expected value. In addition, the coefficients $a(L_i')$ and $b(L_i')$ are substituted with their expected values. This corresponds to computing the expected production cost at the load level L_i'; and

(c) The open loop approach in which the expected value of Λ^* is computed over a truncated Markov model of the generating system described in Appendix II and a small number of load samples generated from the autoregressive model described in Appendix I.

The deterministic case requires the least computational effort while the open loop approach the most. It is worth noting that if in the deterministic case is assumed that $b(L_i') = 0$ (that is the quadratic component of the production cost is ignored) the formulation of this paper is identical to the one of reference [13]. Because in this case the optimization problem defined with (1), (2), (3), (4), and (5) collapses to a linear programming problem.

6. SOLUTION METHOD

The solution method employs a sparcity coded quadratic programming algorithm for the solution of the subproblem 1 and a successive approximations dynamic programming algorithm for the solution of subproblem 2. The combination of these very powerful computational algorithms results in a practical and highly efficient solution method for the problem under consideration. In subsequent paragraphs the computational algorithms are described.

Subproblem 1 Solution Method

Subproblem 1 has been formulated as an optimization problem with quadratic objective function and linear constraints. This problem can be transformed into the following general form:

$$\text{Minimize} \quad c^T z + z^T D z \quad (10)$$

$$\text{Subject to } \quad A_i z \leq b_i$$

$$A_e z = b_e \tag{11}$$

$$z \leq h$$

$$z \leq 0$$

where z is a vector containing the variables x_i and y_i; c, b_i, b_e, and h are vectors of appropriate dimensions; and D, A_i, A_e are matrices of appropriate dimensions. Wolfe [2] has demonstrated that the Simplex Method, appropriately modified can be employed for the solution of above problem. To this purpose, application of Kuhn-Tucker conditions transforms above problem into the simultaneous solution of the following set of equations;

$$\begin{vmatrix} -2D & -A_i^T & -A_e^T & A_e^T & -I & I & 0 \\ A_i & 0 & 0 & 0 & 0 & 0 & I \\ A_e & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} Z \\ \mu' \\ \lambda^+ \\ \lambda^- \\ \mu^2 \\ \nu \\ S^1 \end{vmatrix} = \begin{vmatrix} c \\ b_i \\ b_e \end{vmatrix} \tag{12}$$

$$\begin{aligned} &\nu_i Z_i = 0 && i = 1,2,\dots,n \\ &\mu_j^1 S_j^1 = 0 && j = 1,2,\dots,m \\ &\mu_\ell^2 (h_\ell - Z_\ell) = 0 && \ell = 1,2,\dots,\ell \\ &Z,\ \mu^1,\ \lambda^+,\ \lambda^-,\ \mu^2,\ \nu,\ S^1,\ \ S^2 \geq 0 \end{aligned} \tag{13}$$

The above problem is solved with the basic procedure of phase I of the simplex method as follows:

(a) Introduce artificial variables to construct a starting feasible solution;

(b) Set up a linear program with objective function equal to the sum of artificial variables and constraints (12); and

(c) Utilize the simplex method with upper bounds to solve the problem defined in (b) with the following modifications: A variable cannot enter the solution basis unless it satisfies the exclusivity constraints (13).

A detailed description of the algorithm is given in [3].

The matrix of equations (12) is highly sparse. For this reason the modified simplex method has been sparcity coded. The result is a highly efficient solution algorithm. In this application, repeated solutions of new quadratic problems which result from small perturbations of the defining parameters are required. Sparsity techniques and the dual simplex method enable a fast solution of such problems starting from the solution of a base case. These solutions will be called repeat solutions. The computational requirements of repeat solutions depend on the starting solutions, and number of pumped hydrostorage plants. Repeat solutions have been studied on a CYBER 74 and the following can be stated.

* Average execution time of a repeat solution for a system with one pumped hydrostorage plant is in the order of .05 sec.

* For systems with more than one pumped hydrostorage plants, execution time of a repeat solution increases approximately proportionally to the number of pumped hydrostorage plants.

In summary, the developed sparcity coded quadratic programming is very efficient and suitable for repeat solutions required in the computation of expected values of restitution cost. Detail description and evaluation of the solution method is given in [3].

Subproblem 2 Solution Method:

Subproblem 2 has been formulated as a dynamic program where the state variables are the water level in the upper reservoir of the pumped hydrostorage plants in the beginning of each day. This formulation allows the direct application of the dynamic programming successive approximations technique described in [5] with a dramatic decrease in computational difficulty.

The basic idea of the method is to assume the control variables (defined as the water level in the upper reservoir of the storage plant in the beginning of each day) for all storage plants except one fixed. Then the recurrence formula (8) is defined in terms of one variable only. This recurrence formula is solved to yield the optimal policy of operation for the plant under consideration. The procedure is repeated for every plant. Convergence of this procedure is very fast. As a matter of fact, if the storage plants are ordered in terms of their round trip efficiency, convergence is obtained in one iteration.

7. TEST RESULTS

The developed methodology has been tested and validated with actual data from a local utility. The generating system of this utility is illustrated in Table 1. A typical weekly load curve is illustrated in Figure 3. The autoregressive model representing the load has a random component with a standard deviation of 85 MW. Also Figure 4 illustrates typical curves of the incremental production cost curves, corresponding to the four most probable states of the generating system, namely all units available, unit 11 unavailable, unit 12 unavailable, and unit 8 unavailable, respectively.

Typical policies for the operation of the pumped hydrostorage test system are illustrated in Figure 5. The policies are defined with the target water volume in the upper reservoir in the morning of each day of the planning period which is assumed to be one week. Two policies are depicted in Figure 5. One refers to the deterministic case and the other to the open loop approach. The certainty equivalent approach yields results somewhere in between above cases. Table 2 lists the associated savings resulting from above policies. The dramatic difference between the deterministic case and the open loop approach is due mainly to the dependence of production cost and/or energy replacement cost on the thermal unit availability especially base units. Thus, unit forced outage rates impact greatly on expected production cost savings from storage plants.

Table 1. Test Generating System.

UNIT	CAPACITY (MW)	FORCED OUTAGE RATE	COST COEFFICIENTS B($/MWH)	COST COEFFICIENTS C($/MW/MWH)
1	100.0	[illegible]	19.6332	[illegible]
2	100.0	[illegible]	[illegible]	.046025
3	120.0	[illegible]	[illegible]	[illegible]
4	[illegible]	.0610	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	[illegible]
6	120.0	.0610	[illegible]	[illegible]
7	[illegible]	[illegible]	17.703[illegible]	[illegible]
8	132.0	[illegible]	[illegible]	[illegible]
9	[illegible]	[illegible]	19.7679	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]
11	[illegible]	.2170	[illegible]	[illegible]
12	[illegible]	.2170	[illegible]	[illegible]
13	[illegible]	[illegible]	[illegible]	[illegible]
14	[illegible]	[illegible]	19.1220	[illegible]
15	[illegible]	.0750	19.1220	[illegible]
16	[illegible]	[illegible]	[illegible]	[illegible]
17	[illegible]	[illegible]	[illegible]	[illegible]
18	[illegible]	[illegible]	17.6671	[illegible]
19	40.0	[illegible]	[illegible]	[illegible]
20	40.0	[illegible]	[illegible]	[illegible]
21	[illegible]	[illegible]	[illegible]	[illegible]
22	[illegible]	.0750	[illegible]	[illegible]
23	[illegible]	.0750	[illegible]	[illegible]
24	[illegible]	[illegible]	[illegible]	[illegible]
25	[illegible]	[illegible]	[illegible]	[illegible]
26	[illegible]	[illegible]	[illegible]	[illegible]
27	60.0	[illegible]	[illegible]	[illegible]
28	60.0	[illegible]	[illegible]	.163312
29	[illegible]	[illegible]	[illegible]	[illegible]
30	60.0	[illegible]	19.2931	[illegible]
31	[illegible]	[illegible]	[illegible]	[illegible]
32	[illegible]	[illegible]	[illegible]	[illegible]
33	[illegible]	[illegible]	[illegible]	[illegible]
34	[illegible]	[illegible]	[illegible]	[illegible]
35	[illegible]	[illegible]	[illegible]	[illegible]
36	[illegible]	[illegible]	[illegible]	[illegible]
37	[illegible]	[illegible]	[illegible]	[illegible]
38	[illegible]	[illegible]	[illegible]	[illegible]
39	[illegible]	[illegible]	[illegible]	[illegible]
40	[illegible]	[illegible]	20.3700	[illegible]
41	[illegible]	[illegible]	[illegible]	[illegible]
42	[illegible]	[illegible]	[illegible]	[illegible]
43	[illegible]	[illegible]	[illegible]	[illegible]
44	[illegible]	[illegible]	[illegible]	[illegible]
45	[illegible]	[illegible]	[illegible]	[illegible]
46	120.0	[illegible]	20.8602	[illegible]

CAPACITY 13540

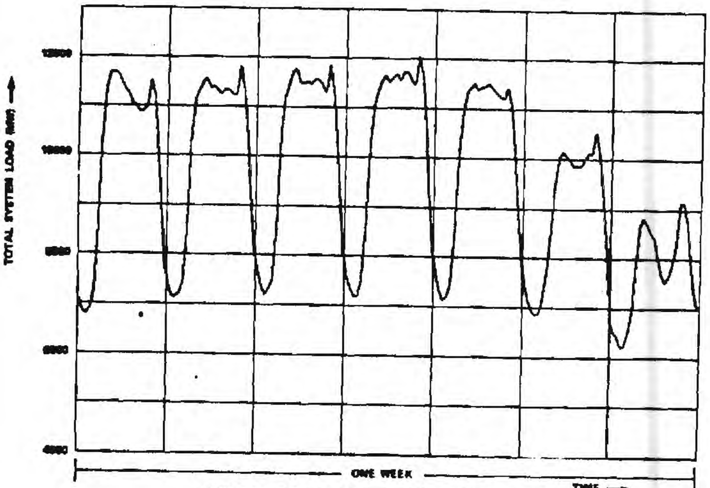


FIGURE 3 A Sample of Chronological Hourly Load Data for the Test System.

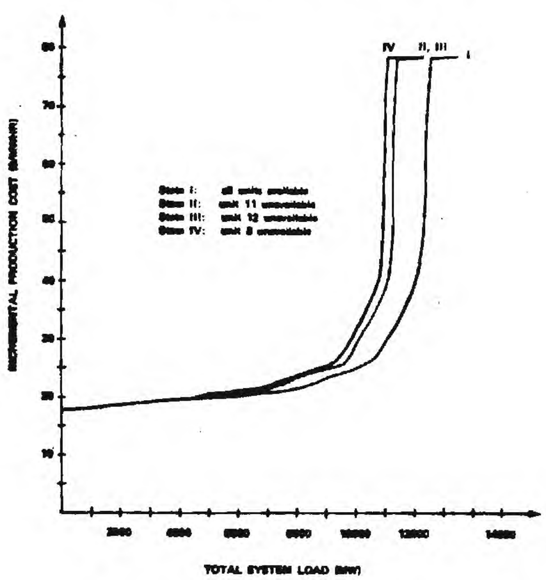


FIGURE 4 Incremental production cost versus total system load for the four most probable states of the generating system.

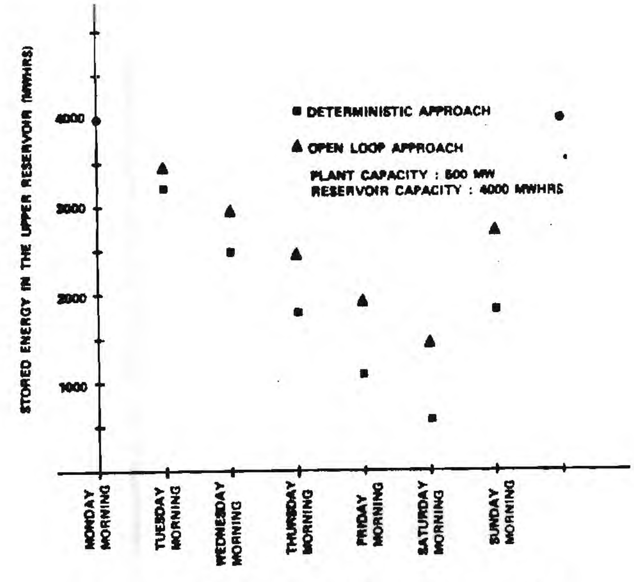


Figure 5. Typical Operation Policies for Pumped Hydrostorage Plants.

Extensive simulation of the test system indicate that optimal economic benefits are attained when the storage plants are operated in such a way that the upper reservoir is full on Monday morning. Thus, optimal operation demonstrates a weekly cycle. To obtain the weekly optimal operation with the methodology of this paper, a planning period of one week must be specified (from Monday morning to Monday morning) and the additional constraint of full upper reservoir at the beginning and end of the week must be imposed. The results of Figure 5 have been obtained in this manner.

Figure 5 illustrates that optimal operation requires that during week days more water should be used for generation than it is pumped back into the upper reservoir. This operation results in a decline of the stored water volume in the upper reservoir in the morning of each day, as is illustrated in Figure 5.

Because of the complexity of the problem we were stimulated to search for suboptimal operating policies which can be easily computed. The results of our investigation indicate that the policy of operating the plant in such a way that the upper reservoir is full in the morning of each day, provides near optimal results. Table 3 lists the associated saving resulting from the mentioned suboptimal operating policy and should be compared with the optimal results of Table 2. The execution time on a CYBER 74 for the results of Figure 5 and Table 2 was 359 seconds while the results of Table 3 were obtained in 49 seconds.

Table 2. Expected Savings Resulting from a 500 MW Storage Plant

Deterministic Approach	: $50,617 per week
Open Loop Approach	: $141,387 per week

Table 3. Expected Savings Resulting from a 500 MW Storage Plant (Suboptimal)

Deterministic Approach	: $49,216 per week
Open Loop Approach	: $136,351 per week

Another study has been made to determine the impact of storage plant rating on the resulting savings. The scenario selected for this study is as follows: Since most modern pumped hydrostorage plants use the same synchronous machine as motor or generator, the pumping or generation capacity was assumed equal. In addition the size of the upper reservoir is assumed to be such that the plant can operate in the generation mode for seven continuous hours starting from full upper reservoir. Under the stated assumptions the weekly savings for various synchronous machine ratings have been computed and plotted in Figure 6. The generating system and load models of the system were kept unchanged. Both deterministic approach and open loop approach were considered. It can be concluded that for a given system there is an optimum value of the capacity of storage plants. Beyond this value the economic effectiveness of storage plants decreases. The impact of uncertainty on the optimal capacity value is substantial. It is also observed that because the deterministic approach tends to underestimate savings resulting from storage plants. This analysis is very important for expansion planning studies. Similar parametric studies can be performed to determine optimal reservoir capacity.

The developed computer program has been employed to investigate the accuracy of linear models reported in [7] and [13] for example. To this purpose the variable $b(L_i)$ in equation (1) was set to zero resulting in a linear program. Comparison of results obtained with the linear and quadratic program reveals the following conclusions: (a) Linear models are accurate for low capacity storage plants (less than 50 MW); and (b) For usual storage plant capacities (around 400 MW) the linear models predict higher expected savings from storage in the order of 30 to 40 per cent.

8. CONCLUSIONS

A two level optimization procedure has been developed and successfully applied to the problem of stochastic coordination of pumped hydrostorage plants with the electric power system. At the first level a quadratic programming optimization procedure is employed to define the optimal scheduling of the storage plant over a period of one day with specified terminal conditions (water level in the upper reservoir at the beginning and end of the day) At the second level a dynamic programming successive approximations procedure is employed to define optimal operating policies defined with target water volume in the upper reservoir at the beginning of each day. At this level uncertainty associated with generating unit availability and electric load level is taken into consideration. The generating sytem is represented with a truncated Markov model and the hourly electric load is represented with an autoregressive model.

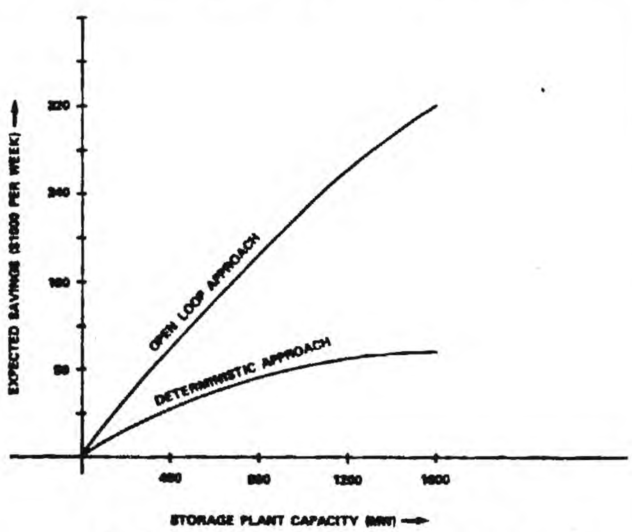


Figure 6. Expected Savings Due to Storage Plants as a Function of Plant Capacity.

A computer program has been developed based on the described methodology. The computer program has two options. Option one assumes that the generating system and electric load are deterministically known. Option two employs the mentioned stochastic models of the generating system and electric load. The computer program has been employed to obtain representative results with a test system. The results indicate that:

(1) The potential savings resulting from storage plants are usually underestimated with deterministic models.

(2) For a given electric power system there exists an optimum capacity of storage plants. Beyond this capacity the economic effectiveness of storage is deteriorated.

(3) The impact of thermal unit forced outages and electric load uncertainty is substantial.

(4) The policy of having full upper reservoir at daybreak yields near optimal results.

ACKNOWLEDGEMENTS

The authors express their appreciation to the National Science Foundation for their support of the research reported in this paper (NSF Grant #ENG 7816496).

9. REFERENCES

1. A. P. Meliopoulos, "A Decomposition Technique to Determine Optimal Coordinating Policies of Energy Storage Plants With Electric Power Systems," Proceedings of the 18th IEEE Conference on Decision and Control, pp. 876-878, Fort Lauderdale, FL, Dec. 12-14, 1979.

2. P. Wolfe, "The Simplex Method for Quadratic Programming," Econometrica, vol. 27, 3, pp. 382-298, July 1959.

3. A. P. Meliopoulos, "Optimal Coordination of Energy Storage Facilities With an Integrated Electric Power System," Progress Report, NSF Grant No. ENG7816496.

4. S. Vemuri, E. F. Hill, and R. Balasburamanian, "Load Forecasting Using Stochastic Models," paper presented at the 1976 IEEE Winter Meeting, New York, NY, Feb. 1976.

5. R. E. Larson, A. J. Korsak, "A Dynamic Programming Successive Approximations Technique with Convergence Proofs," Automatica, vol. 6, pp. 245-252, 1970.

6. A. F. Glimn and L. K. Kirchmayer, "Economic Operation of Variable Head Hydroelectric Plants," IEEE Trans. Power App. Syst., vol. 77, Dec. 1958, pp. 1070-1079.

7. P. J. Bernard, J. F. Dopazo, and G. W. Stagg, "A Method of Economic Scheduling of a Combined Hydro and Steam Generating System," IEEE Trans. PAS, vol. 83, Jan. 1964, pp. 23-30.

8. G. H. McDaniel and A. F. Gabrielle, "Dispatching Pumped Storage Hydro," IEEE Trans. PAS, vol. 85, 1966, pp. 465-471.

9. Survey of Pumped Storage Projects in the United States and Canada to 1975, IEEE Committee Report, presented at the 1976 Winter Power Meeting.

10. "Hydro-Electric Power Resources of the United States: Developed and Undeveloped," Document No. 015-000-00347-2, 1976.

11. C. D. Galloway and R. J. Ringlee, "An Investigation of Pumped Storage Scheduling," IEEE Trans. PAS, vol. 85, no. 5, May 1966, pp. 459-465.

12. F. A. Viramantes and H. B. Hamilton, "Scheduling of Hydro Electric Storage Plants Using Dynamic Programming," paper presented at the IEEE PAS Summer Meeting, Mexico City, Mexico, July 17-22, 1977.

13. C. Dechamps, R. Muytten, and S. T. Lee, "Optimal Operation of Storage Devices in a Power System," paper A80-085-1 presented at the IEEE-PES Winter Power Meeting, New York, NY, Feb. 3-8, 1980.

14. A. E. Allen, "Potential for Conventional and Underground Pumped-Storage," IEEE Transactions on Power Apparatus and Systems PAS, vol. 96, no. 3, pp. 993-998, May/June 1977.

15. P. H. Henault, "On the Application of Dynamic Programming to Long Range Planning with an Uncertain Future," Ph.D. Dissertation, Department of Industrial Engineering at Stanford University, 1969.

APPENDIX I

Electric Load Model

This Appendix provides a description of the stochastic model employed for the electric load. The described model has been selected after a thorough investigation and evaluation of existing models. The model employed has been taken from reference [4] and it is briefly described in this Appendix.

Analysis of historical data of electrical load, L_t, where L_t is the load level at time t, indicates that the electric load is a nonstationary stochastic process. Modeling and identification of nonstationary stochastic processes is quite difficult. Fortunately, it has been observed [4] that an appropriate differential of the nonstationary stochastic process L_t may behave as a weakly stationary stochastic process. Also, because the electric load, L_t, exhibits daily, weekly, and seasonal periodicity it is expedient to define a transformation of L_t into an assumed weakly stationary stochastic process x_t, as follows:

$$x_t = \nabla^d \nabla_S{}^D L_t \tag{I.1}$$

where

$\nabla = 1 - B$ is a nonseasonal backward difference operator

$\nabla_s = 1 - B^s$ is a seasonal backward difference operator

d is the degree of nonseasonal differencing

B is the backward operator defined with $BL_t = L_{t-1}$

D is the degree of seasonal differencing.

A stationary stochatic process, such as x_t, can be generated if white noise is supposed to be transformed by a linear filter. In the simplest case the linear filter can be an identity transformation, resulting to an autoregressive model for the load as follows:

$$L_t = \sum_{i=1}^{n} a_i L_{t-\zeta(i)} + \eta_t \tag{I.2}$$

where the white noise, η_t, is defined as follows:

$$E\{\eta_t\} = 0$$

$$\mathrm{Cov}\{\eta_t, \eta_{t+\tau}\} = \begin{cases} \sigma_\eta^2 & \text{if } \tau=0 \\ 0 & \text{otherwise} \end{cases}$$

The defined model involves n parameters a_i, i=1, ..., n, which are estimated via least square estimation. Hourly load data for several weeks are required to carry out the least square estimation of the parameters a_i, i=1,2,...,n. Once the model has been identified, it is evaluated to determine level of uncertainty. This is done by computing the standard deviation of the prediction. If the level of uncertainty is acceptable, the model is complete. Otherwise, another model has to be selected and the procedure is repeated [4].

Analysis of actual utility data indicate that a low order model provides good fit to historical data. For the test system a seven order model was employed. The historical data used were obtained from the local utility.

The developed stochastic model is employed to generate samples of future electric loads. To this purpose a random number generator is employed to generate the noise η_t. Projected hourly loads L_t are generated by applying formula (I.2). The weekly load curve of Figure 3 has been generated from the model described here.

APPENDIX II

This Appendix describes the stochastic model for the computation of incremental production costs of an electric power system. A two state (up - down) Markov model is employed for the representation of a generating unit. It is assumed that transitions from one state to the other occur only in the beginning of a day. Thus for the duration of one day the set of units which are in the up state remain the same. This set is symbolically represented with G_A. Once the set of available generating units, G_A, is known the incremental production cost at a given total system load is computed via a bus-bar economic dispatch. The probability of existence of the computed incremental production cost equal the probability of existence of state G_A. Subsequently, the probability of existence of state G_A is described as well as the computation of the incremental production cost for a given set G_A.

Probability of Existence of State G_A: The set G_A by definition comprises m generating units taken from a generating system of n units. In general $n \geq m$. Let

$P_i^{(k)}$ be the probability that unit i is available during the day k.

λ_i be the conditional failure rate of unit i expressed in per day units.

μ_i be the repair rate of unit i expressed in per day units.

Then

$$P_i^{(k+1)} = P_i^{(k)} (1-\lambda_i) + (1 - P_i^{(k)})\mu_i \qquad i=1,2,\ldots,n \tag{II.1}$$

Given the state of a generating system at the beginning of a planning period

$$(P_i^{(0)}, \quad i = 1,2, \ldots, n)$$

the probability of availability of every unit at day k is obtained by way of numerical solution of equations (II.1).

Now consider state G_A. The probability of existence of state G_A during day k denoted with $Pr[G_A^{(k)}]$ will be

$$Pr\ G_A^{(k)} = \prod_{i\epsilon\{G_A\}} P_i^{(k)} \prod_{j\notin\{G_A\}} (1 - P_j^{(k)}) \tag{II.2}$$

Computation of the Incremental Production Cost: The incremental production cost at a given total load is computed with the aid of a bus-bar economic dispatch. The problem is formulated as follows:

$$\text{Minimize} \quad C_o = \sum_{i=1}^{m} f_i(u_i)$$

$$\text{subject to} \quad \sum_{i=1} u_i - L' = 0$$

$$\underline{u} \leq u_i \leq \overline{u}_i \qquad \text{or } u_i = 0 \qquad i = 1,2, \ldots, m$$

where

u_i is output (MW) of unit i

L' is the total adjusted load of the thermal generating units (equals electric load plus losses and plus or minus the power input or output of all pumped hydrostorage plants)

$\underline{u}_i, \overline{u}_i$ minimum and maximum allowable operating limits of unit i.

The solution of the above problem is a function of total adjusted load L':

$$C_o^* = F(L')$$

The function F is obtained with a repetitive solution of the defined problem at various values of L'. To this purpose the sparcity coded quadratic program is employed which is described in this paper.

The incremental production cost of the system at total adjusted load L' is

$$g(L') = \frac{dF(L')}{dL'}$$

The function g(L') is not smooth. However, smoothing techniques are applied to yield a smooth function. Typical curves are illustrated in Figure 4.

A computer program has been developed to perform the above computations. The program will yield 20 incremental production cost curves corresponding to the 20 most probable states of the System of Table 1 in less than one minute execution time on a CYBER 74.

COMPUTER AIDED INSTRUCTION OF ENERGY SOURCE UTILIZATION PROBLEMS

A. P. Meliopoulos

1. Abstract

Energy usage in the form of electricity constitutes a large portion of energy consumption in developed countries. Electric energy is generated from primary energy sources such as coal, nuclear, natural gas, petroleum, hydro, geothermal, etc. The generation, management and utilization of electric energy is constrained with technological factors, environmental and safety considerations, capital requirements, etc., resulting in limitations of primary energy source utilization. This paper describes an interactive computer program which has been developed to aid the instruction of problems related to primary energy source utilization for the production of electric energy. The computer program has been developed for a graduate course on power system planning. The program can be used to study the major implications of alternative national energy policies, as well as utility originated policies. Examples are: load or supply management, dispersed or central storage, dispersed generation, cogeneration, plant conversion, etc. The paper describes the interactive simulation program and associated theory and three examples of computer program utilization.

2. INTRODUCTION

Modern societies consume large amounts of energy in the form of electricity. No one disputes the practicality and comfort of electricity. However, the accepted practice necessitates the generation of electricity from primary energy sources such as coal, nuclear, gas, hydro, geothermal, wind, etc. The utilization of primary energy sources to produce electricity is constrained by numerous factors such as:

* Technology
* Availability of Funding
* Safety Considerations
* Availability of primary energy sources
* Operating Constraints of Power Plants
* Long Construction Times for Power Plants
* Pollution Constraints, etc.

Long range planning of power systems is essential in the implementation of a sound energy policy. Stated in a different way a national energy policy should guide electric power utilities to expand or convert their facilities in such a way as to achieve the stated objectives of the policy. A very popular objective, for example, will be to decrease petroleum utilization for the production of electric energy. Irrespectively of the specific objectives, central to any policy formulation or planning decision is the accurate (as much as possible) projection of primary energy source consupmtion for a given set of data and assumed future scenarios.

In its simplest form this problem can be stated as follows: For a projected economic environment and projected needs for electric energy, estimate the required amounts of primary energy sources. Because of physical restrictions and the objective of electric utilities to maximize their profits the conputation of above estimate is complex. Ramifications to above problem can be the estimation of the amount of environmental pollution, the computation of the impact of new trends such as load/supply management, cogeneration, etc.

The study of the mentioned complex problems in an academic environment is rather difficult. To aid students, who may very well be the energy policy makers of tomorrow, to understand the limitations in utilizing primary energy sources and estimate the impact of new trends or policies, an interactive power system simulation computer program has been developed. The program is utilized by students to study through homework assignments the major characteristics of the mentioned problems. These studies are part of a one academic quarter course on Power System Planning (EE 6501). This paper describes this educational tool and illustrates its utilization for energy related studies.

In subsequent paragraphs a description of the power system simulation procedure will be given. Then the interactive computer program and its utilization will be described. The paper will include examples which demonstrate the utilization of the interactive computer program.

3. POWER SYSTEM SIMULATION

The problem of electric power system simulation may be defined as follows: Given the forecasted electric load demand for the time period under consideration and a list of available generating units of the system, simulate the operation of the system in order to forecast energy generated by units, cost and required fuel, taking into account the effects of scheduling functions within the time period considered and the random forced outages of the units.

In recent years a very successful probabilistic simulation technique has been introduced [3], [4]. This technique will be briefly discussed.

The point of departure for the methodology is the development of probabilistic models for generating units and the electric power demand. The probabilistic model of a generating unit of capacity C MW is shown in Figure 1. This is known as the up and down state model. This model can be interpreted as follows: With probability p the unit is available for production and with probability q=1-p is unavailable. The quantity q is known as the forced outage rate of the unit (F. O. R.)

The electric power demand probabilistic model is developed as follows: The electric power demand of a system is plotted as a function of time to obtain the chronological load demand curve. From the chronological load curve, another curve can be constructed which describes the length of time for which the load was greater than a specified value. In addition the time axis is normalized by the duration T of the time period considered and it is shown in Figure 2. The generated curve can be interpreted as follows. Consider point A on the curve with coordinates $\frac{t}{T}$, ℓ. Over the period T the load on the system is greater than ℓ for a time period equal to t. Therefore, if the frequency theory of probability is used and the load duration curve is referred to the future, the following definition follows:

$$\Pr\ [\text{load} \geq \ell] = \frac{t}{T}$$

It is apparent that the normalized time axis can be interpreted as probability. If $L(\ell)$ denotes the normalized inverted load duration curve the following relationship will hold:

$$L(\ell) = \Pr\ [\text{load} \geq \ell]$$

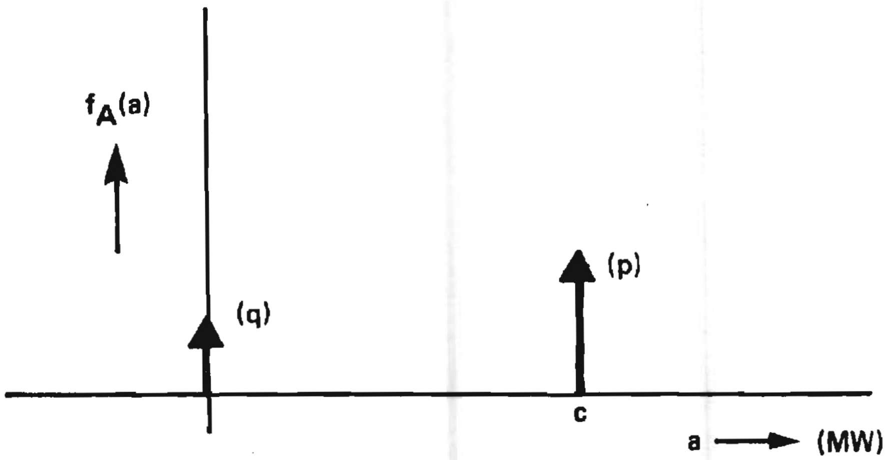


Figure 1. Probability Density Function of the Available Capacity of a Generating Unit.

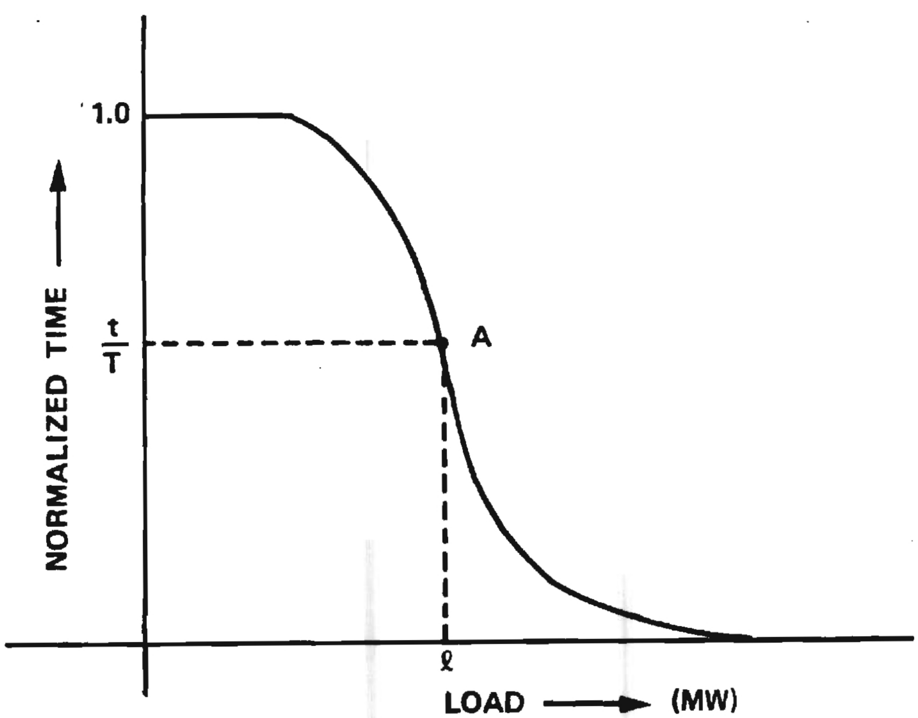


Figure 2. Normalized Inverted Load Duration Curve

The function $F_L(\ell)$,

$$F_L(\ell) = \Pr\,[\text{load} \leq \ell] = 1.0 - L(\ell)$$

defines the probability distribution function (or cummulative probability function) of the variable ℓ, the total load on the system, which is interpreted as a random variable.

The functions $L(\ell)$ and $F_L(\ell)$ describe the same thing, the statistics of the variable ℓ. The function $F_L(\ell)$ is the usual probability distribution function while $L(\ell)$ is called the inverted probability distribution function (IPDF).

Given the above models the following quantities can be computed:

1. Probability of operation of unit 1:

$$\Pr\,[\text{Unit 1 in operation}] = \Pr\,[\text{Unit 1 Output} > 0]$$

2. Expected value of produced energy from the unit.
3. Expected value of cost of operation of unit i.
4. Expected fuel amount required of unit i.
5. Expected pollutant amounts to be emitted from unit i.

The forementioned quantities can be computed as follows:

Consider that the n-units of the system operate at level x_1, x_2, ..., x_n. If unit k is not in operation, then obviously x_k will equal 0. Since there is a finite probability that any unit can be forced out, the output of unit j, x_j, can be considered to be a random variable with probability of forced outage equal to q_j. We write

$$\Pr\,(A_j = x_j) = 1 - q_j \quad , \quad x_j \neq 0$$
$$\Pr\,(A_j = 0) = q_j$$

where A_j is a random variable representing the available capacity of unit j. Above relationships state that the probability that the output of generator j is x_j equals $1-q_j$, and the probability that the same output is zero equals q_j.

Further assume that the electric load equals ℓ. The probability distribution function is determined with the load duration curve:

$$F_L(\ell) = 1.0 - L(\ell)$$

For the condition that has been considered, the apparent load, ℓ_a, will be

$$\ell_a = \ell - x_1 - x_2 - \dots - x_n$$

Since $\ell, x_1, \dots x_n$ are not deterministically known, the above equation can be replaced with its equivalent equation in terms of the corresponding random variables:

$$L_a = L - A_1 - A_2 - \dots - A_n$$

where L is a random variable representing the electric load and A_i is a random variable representing the output of unit i. Since the probability distribution functions of the random variables L, A_1, ..., A_n are known and since these random variables are independent, the probability distribution function of the random variable L_a can be easily computed with a series of convolutions [1], [3], [4].

If we assume that $\ell_a > 0$ (that is, load exceeds generation) then another unit should be brought into operation or one or more of the operating units should increase their output. Assume that unit i is operating at x_i and that it is selected according to an arbitrary dispatch criterion to respond to any increases in the load. This arbitrary dispatch criterion will be qualified later. Without loss of generality x_i may be equal to zero. In general, if $\ell_a > 0$ the output of unit i will increase from x_i to $x_i + \Delta x_i$ where Δx_i is a small increment (1 to 2 MW). We shall refer to this increment as the block Δx_i. It should be obvious that if $x_i = 0$, the increment Δx_i may not be small. In this case unit i will be brought into operation at a level at least equal to minimum allowable operating level. With the described formulation and application of basic probability theory, it is easy to derive expressions for the expected energy to be produced, cost of operation, required fuel, and pollutant emission from the Δx_i increase in the output of generator i. The results are summarized as follows:

Step 1. Compute the probability distribution function of random variable

$$L' = L_a + A_i$$

let it be $F_{L'}(z)$.

If $x_i = 0$, skip this step and assume $F_{L'}(z) = F_L(z)$

Step 2. Compute:

$$E(\Delta x_i) = (1-q_i)T \int_{z=x_i}^{x_i+\Delta x_i} (1-F_{L'}(z))dz \qquad (1)$$

$$C(\Delta x_i) = (1-q_i)T\ f_i(o)(1-F_{L'}(o))\delta(x_i) +$$

$$+(1-q_i)T \int_{z=x_i}^{x_i+\Delta x_i} \frac{df(z)}{dz} (1-F_{L'}(z))dz \qquad (2)$$

where $\delta(x_i) = \begin{cases} 1 \text{ if } x_i=0 \\ 0 \text{ if } x_i\neq 0 \end{cases}$

$f(z)$ is the heat curve of unit i

$\frac{df(z)}{dz}$ is the incremental heat rate of unit i

$E(\Delta x_i)$ = expected energy to be produced from block Δx_i

and $C(\Delta x_i)$ = expected heat consumed by block Δx_i

Similarly plant pollution output can be computed. If the pollution curve for the plant is known as a function of power output, $g(z)$, the expected amount of pollutants to be produced from block Δx_i will be:

$$G(\Delta x_i) = (1-q_1)Tg(o)(1-F_{L'}(o))\delta(x_i)$$

$$+(1-q_1)T \int_{z=x_i}^{x_i+\Delta x_i} \frac{dg(z)}{dz}\,(1-F_{L'}(z))dz. \qquad (3)$$

where $G(\Delta x_i)$ = expected amount of pollutants from block Δx_i

The cost of operation of block Δx_i is computed from

$$\text{Cost}(\Delta x_i) = C(\Delta x_i)\,\frac{p}{h}$$

where p price (\$/kg) of fuel

h heat content of fuel (cal/kg)

The fuel required for the operation of block Δx_i is computed from

$$\text{Fuel}(\Delta x_i) = C(\Delta x_i)/h$$

Once the above computations have been completed the apparent load, after loading of block Δx_i, will be

$$\ell_a = \ell - x_i - x_2 - \ldots - (x_i + \Delta x_i) - \ldots - x_n$$

The probability distribution function of L_a can be easily computed if we regocnize that

$$\ell_a = \ell' - (x_i + \Delta x_i)$$

In this case a convolution between the probability density function of variable L' with the probability density function of variable $x_i + \Delta x_i$ will suffice to determine the statistics of variable L_a.

The application of above procedure for the computation of expected production quantities is obvious. Each generating unit is partitioned into a number of blocks. The procedure is then applied on each one of the blocks.

4. SIMULATION OF DISPATCH PRACTICES

The described procedure, which can be easily implemented, simulates incremental dispatch practices since the incremental loading of the units is based on an arbitrary criterion. Therefore, the presented method is capable of simulating any dispatch practice.

In general an arbitrary criterion for unit dispatching may take the form of a nonlinear "cost" function of power plant output as it is shown in Figure 3. The "cost" may be actual fuel cost, a penalty function for pollution emission, weighted combination of both or any other function. Depending on the selection of the "cost," alternative dispatch procedures may be simulated.

Appropriate selection of the "cost" function can effect the simulation of alternative dispatch practices resulting from governmental regulations for example. Reference 2 describes how this concept can be utilized to simulate alternative dispatch practices to meet pollution constraints.

In general electric power systems operate on the basis of minimum cost. In this case the "cost" function should be selected to be the actual fuel cost curve for the power plants (economic dispatch).

5. DESCRIPTION OF THE INTERACTIVE COMPUTER PROGRAM

An interactive computer program has been developed based on the described electric power system simulation procedure. No computer experience is required to use the interactive program. To this purpose a complete instruction set has been included in the interactive mode which guides the user in a step by step procedure for the system data definition, simulation and result analysis.

The interactive computer program is partitioned into 5 modules which are controlled by a master directory. The selection of any one of the modules can be effected by typing an appropriate code. Code description is communicated to the user. For example upon initiation of the interactive program execution the user will observe on the teletype screen the message of Figure 4. Upon selection of a module by typing the appropriate code the program will respond with specific questions to which the user should answer. The user is always given the option to return to the master control.

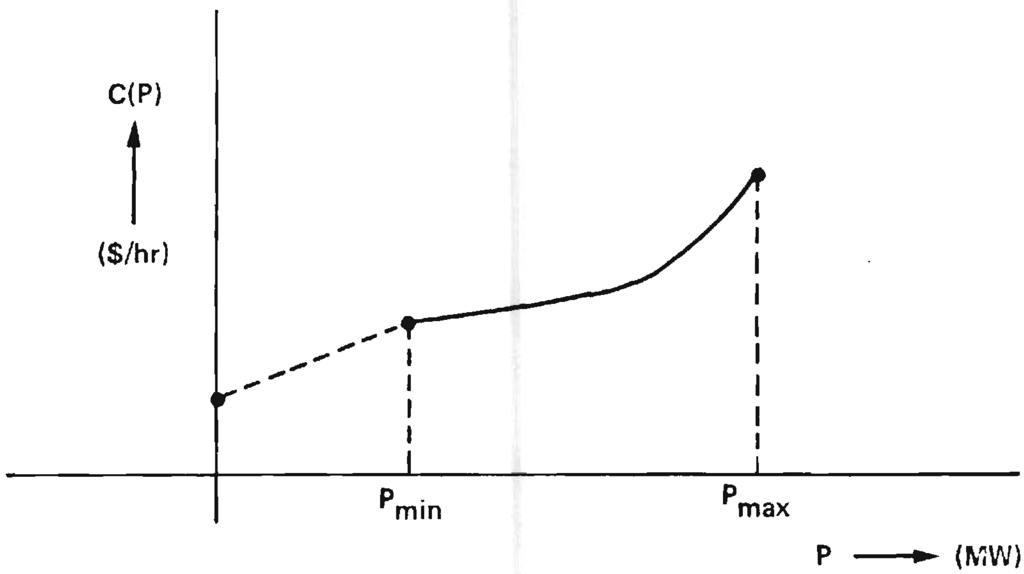


Figure 3. "Cost" Criterion for Generation Dispatch

Modules 1, 2 and 3 are utilized to define system data. The user is asked to prepare data similar to those of Table 1, Figure 5 and Table 2 for the generating units, electric load and fuel respectively. The normalized inverted load duration curve is defined with the coordinates of a number of points as it is illustrated in Figure 5. The user should use his judgement to select a number of points which will adequately represent the continuous inverted load duration curve. The interactive modules 1, 2, 3 will ask questions to which the user should respond. In this way the generating system, load and fuel data are fed into the computer. The user is given the option of inspecting or modifying the data while in modules 1, 2 or 3.

The simulation module 4 should be called upon only when the generation system, load and fuel data have been defined. Upon completion of the simulation the program will return to the master control. At this point the user may select module 5 to inspect the simulation results. These results consist of consumed fuel by unit and total, production cost by unit and total, generated pollution by unit and total, energy production by unit and total and system loss of load probability. The last is defined as the probability that the electric load will exceed generation.

THIS IS THE MASTER CONTROL

MODES OF INTERACTION

M O D E	C O D E
GENERATION DATA INPUT/MODIFICATION	1
LOAD DATA INPUT/MODIFICATION	2
FUEL DATA INPUT/MODIFICATION	3
SIMULATION	4
RESULTS INSPECTION	5
TERMINATION	6

TO SELECT A MODE TYPE CORRESPONDING CODE

?

Figure 4. Message Communicated to the User by the Interactive Power System Simulation Program.

TABLE 1 GENERATION SYSTEM DATA

UNIT	TYPE	CAPACITY (MW)	F.O.R.*	A (10^9 cal/hr)	B (10^9 cal/MWHR)	C (10^9 cal/(MW)^2/HR)
1	NUCLEAR	600	.20	125	2.1	.0003
2	COAL	150	.10	45	1.8	.000255
3	COAL	150	.10	45	1.8	.000255
4	COAL	150	.10	45	1.8	.000255
5	PETROLEUM	100	.06	2.217	1.685	.000044
6	PETROLEUM	100	.06	2.217	1.685	.000044
7	NATURAL GAS	50	.03	2.1	1.62	.000042
8	NATURAL GAS	50	.03	2.1	1.62	.000042

*F.O.R. = Forced Outage Rate

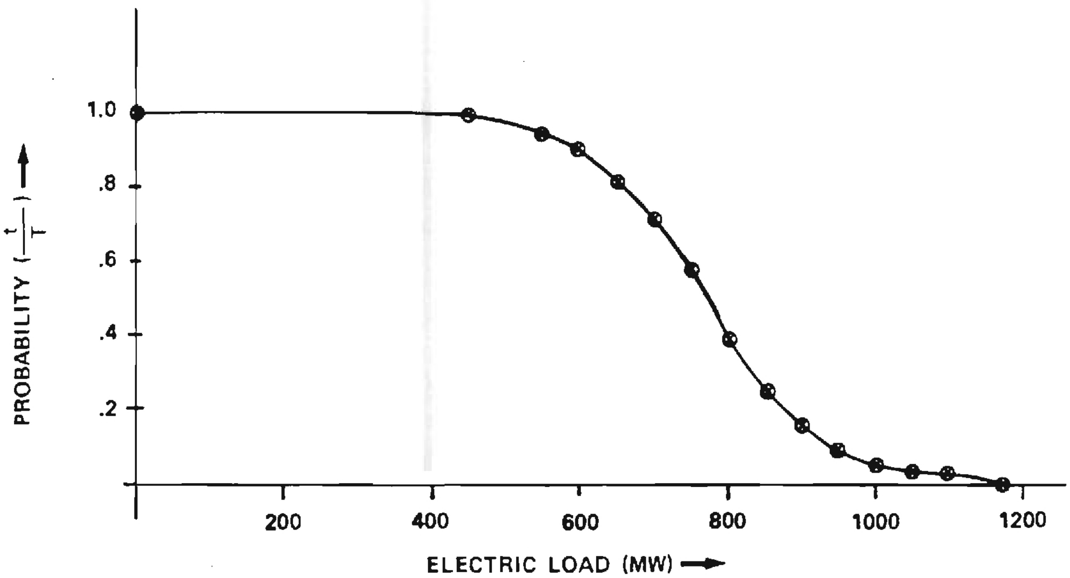


Figure 5. Normalized Inverted Load Duration Curve for the Test System.

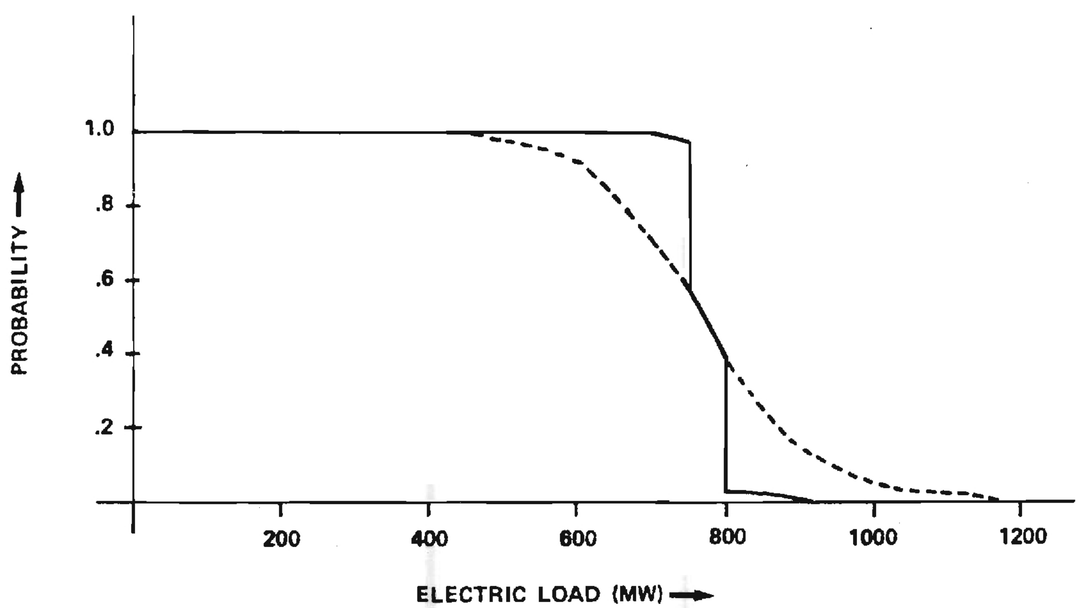


Figure 6. Normalized Inverted Load Duration Curve for the Test System with a Pumped Hydrostorage Plant.

TABLE 2 FUEL DATA

Fuel Type	Heat Content Kcal/Kg	Price $/Kg
Nuclear	19.069×10^9	35,000.
Coal	6000.	.04
Petroleum	10200.	.23
Natural Gas	210000.	5.0

6. INTERACTIVE PROGRAM UTILIZATION

The interactive power system simulation program has been specifically developed to support instruction of a graduate course on power system planning. It can be utilized to study numerous problems related to power system planning such as:

1. Production Costing
2. Project fuel requirements
3. Analyze power interchange agreements among power companies
4. Effects of load management
5. Effects of central storage plants
6. Effect of governmental regulations for pollution control
7. Effects of cogeneration
8. Effects of conversion of power plants from one fuel to another.

Above problems can be quantitatively studied in the framework of assigned homework with the aid of the interactive computer program. Because of space limitations it is rather difficult to discuss in detail computer program utilization with respect to each one of the above problems. For the purposes of this paper, computer program utilization will be demonstrated with three simplified examples. The objective of these examples will be limited to study the effects of

a) Central Storage Plants
b) Cogeneration
c) Conversion of Power Plants from one Fuel to Another

on primary energy source utilization. To this purpose a hypothetical electric power utility with nuclear, coal, petroleum and natural gas as primary energy sources will be assumed. Table 1, Figure 5 and Table 2 define all pertinent data for the test power system. The weekly consumption of primary energy sources computed with the interactive program is illustrated in Table 3. In addition average cost of produced electric energy and system loss of load probability is listed. This data will be considered as the base case data. In subsequent paragraphs the impact of central storage, cogeneration and plant conversion on primary energy source utilization by the assumed test system will be examined.

6.1 Impact of Central Storage

Central storage plants are currently being used to enable supply management. Present economics justify pumped hydrostorage plants.

Pumped hydrostorage plants consist of two water reservoirs located at different altitudes. During night hours the electric load on a system is low. Base power plants (nuclear, coal) with low production cost suffice to meet the demand. During night hours pumped hydrostorage plants use the inexpensively produced electricity to pump water in the upper reservoir. Later in the day as the electric load increases, the stored water in the upper reservoir will be utilized to produce electricity in such a way as to minimize operation of power plants burning expensive fuels such as natural gas, petroleum, etc.

The purpose of this example is to demonstrate the effect of central storage plants on primary energy source utilization. It will be hypothesized that a central storage plant of 250 MW capacity and an average daily pumping duty of 900 MWHRS is added to the test power system.

TABLE 3 WEEKLY PRODUCTION QUANTITIES FOR THE TEST SYSTEM

NUCLEAR FUEL	10.424 kg
COAL	12.3495×10^6 kg
PETROLEUM	1.5482×10^6 kg
NATURAL GAS	27644 kg
AVERAGE COST	10.726 \$/MWHR
LOLP	.142648

It can be argued that the inverted load duration curve will be modified as in Figure 6. This is a simplification but very close to reality. The interactive computer program can be utilized to project primary energy source utilization with or without central storage. Table 4, column A, illustrates the results. Obviously use of central storage for the test system increases utilization of coal (assuming availability) while decreases demand for petroleum and natural gas. In addition average production cost decreases and system reliability deteriorates.

6.2 Cogeneration

Many industries require steam for their processes. Normally low pressure, low temperature steam will suffice. The same industry may also need electric energy. In many cases it is economically expedient to produce high pressure, high temperature steam and to route this steam through a high pressure steam turbine coupled with a generator. In this way the steam coming out of the turbine is conditioned for the needs of the industry while at the same time electric power is produced. The generated electric power may be partly consumed by the industry and the remaining returned to the electric power grid. This practice has been common in Europe and limited in the U. S. Recent efforts tend to spread the use of cogeneration in the U. S.

Simulation of cogeneration to project the impact on primary energy source utilization is quite complex. A simplified approach can be justified as follows: A cogenerating plant operates beyond the control of the power company. If it is assumed that the participating industries operate three shifts daily then the cogenerating plant will operate continuously assuming it is available. This operation can be simulated by modeling the cogenerating plant as another power plant with a certain forced outage

Table 4. Effects of Central Storage, Cogeneration and Plant Conversions on Primary Energy Source Utilization.

	PERCENT CHANGE FROM BASE CASE		
	A	B	C
	250 MW CENTRAL STORAGE	COGENERATION	CONVERSION OF UNIT # 6 INTO COAL
NUCLEAR	.95%	–.93%	0%
COAL	10.32%	–12.96%	15.30%
PETROLEUM	–20.40%	–10.81%	–55.97%
NATURAL GAS	–5.97%	–7.43%	.80%
AVERAGE PRODUCTION COST	–3.06%	–8.72%	–8.98%
LOLP	38.12%	–19.13%	1.2%

rate (FOR) and zero "cost" to the power company. Then the power system will be simulated with or without cogeneration and the results compared. Table 4, column B illustrates the results for an assumed cogenerating plant of 50 MW. In these results the primary energy source required by the cogenerating plant is not taken into account. It is observed that the power system consumes less amounts of primary energy sources. The percentage change is higher for less expensive fuels (nuclear, coal) and lower for high grade guels. Also average production cost decreases but so do the company's revenues because of lower energy sales. A better picture of the effects of cogeneration is obtained if the primary energy source of the cogenerating plant and the sales agreement between the power company and the cogenerating plant are considered. This discussion is beyond the scope of this paper.

6.3 Conversion of Power Plants from One Fuel to Another

In this example it will be hypothesized that one generating plant will be converted from a petroleum burning plant into a coal burning plant. In particular plant #6 (Table 1) will be converted. Utilization of the interactive simulation program will project fuel requirements for the system. These requirements are compared with the requirement of the original system in Table 4, column C. Petroleum needs are reduced by 55%, natural gas needs remain practically the same while coal needs increase by 15.3%. The average production cost decreases by 8.98%, and system reliability remains practically unchanged. Plant conversion seems to be attractive from above observations. However two other additional considerations need to be examined. First the plant conversion cost and second the pollution problem associated with coal units. Discussion of these problems are beyond the scope of this paper.

7. CONCLUSIONS

The analysis of primary energy source utilization by electric power systems is a copmlex problem but necessary in assessing energy needs. To aid instruction of related problems an interactive electric power system simulation program has been developed. This program is utilized in a one quarter graduate course on power system planning to provide analytical support for the study of energy related topics. Specifically limitations in primary energy source utilization by power systems and the impact of trends or policies can be quantitatively studied with the aid of the interactive program. The paper presents three examples of program utilization.

ACKNOWLEDGEMENTS

The work reported in this paper has been partially funded by the National Science Foundation (NSF Grant #ENG-7816496).

8. REFERENCES

1. A. P. Meliopoulos, Power System Planning - Class Notes.

2. A. P. Meliopoulos, "Impact of Constrained Pollution Dispatch Practices on Electric Power Production Costs", paper submitted for presentation at the 1981 IEEE-PES Winter Meeting, to be held in Atlanta, Georgia, February, 1981.

3. R. R. Booth, "Power System Simulation Model Based on Probability Analysis", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-91, No. 1, Jan./Feb. 1972.

4. M. A. Sager, et al., "A New Generation Production Cost Porgram to Recognize Forced Outages", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-91, Sept./Oct. 1972.

VI. BIBLIOGRAPHY

BIBLIOGRAPHY

Scheduling Methodologies

1. A. F. Glimn and L. K. Kirchmayer, "Economic Operation of Variable-Head Hydroelectric Plants", Trans. AIEE (Power Apparatus and Systems), vol. 77, December 1958, pp 1070-1079.

2. C. D. Galloway and R. J. Ringlee, "An Investigation of Pumped Storage Scheduling", IEEE Transactions on Power Apparatus and Systems, vol. PAS-85, No. 5, May 1966, pp 459-465.

3. T. Kennedy and E. M. Mabuce, "Dispatch of Pumped Hydro Storage on an Interconnected Hydro-Thermal System", IEEE Transactions on Power Apparatus and Systems, vol. PAS-84, 1965, pp 446-457.

4. R. J. Ringlee, "Economic Scheduling and Automatic Dispatching of a Hydro-Thermal Power System", paper presented at the 1964 American Power Conference, Chicago, IL.

5. M. J. Cobian, "Optimal Pumped Storage Operation with Interconnected Power Systems", IEEE Transactions on Power Apparatus and Systems, vol. PAS-90, May/June 1971, pp 1391-1399.

6. C. D. Galloway, I. H. Landes, and W. D. Marsh, "The Role of Pumped Storage in Generation Systems", IEEE Transactions on Power Apparatus and Systems, 1965 or 1966.

7. L. Paris and L. Salvaderi, "Optimization of the Basic Characteristics of Pumped Storage Plants", Presented at the 1974 IEEE Winter Power Meeting.

8. R. J. Cypser, "The Optimum Use of Water Storage in Hydro-Thermal Electric Systems", Sc.D. Thesis, Massachusetts Institute of Technology, Cambridge, MA, January, 1953.

9. O. Coskunoglu, "A Decentralized Control Strategy with an Application to the Regulation of a Set of Water Reservoirs", Proceedings of the 19th IEEE Conference on Decision and Control, Albuquerque, NM, December 10-12, 1980, pp 166-167.

10. F. J. Rees and R. E. Larson, "Computer Aided Dispatching and Operations Planning for an Electric Utility with Multiple Types of Generation", IEEE PAS-90, March/April, 1971, pp 891-899.

11. H. W. Kuhn and A. W. Tucker, "Nonlinear Programming", Proceedings 2nd Berkeley Symposium on Mathematical Statistics and Probability (1950), Berkeley, California: University of California Press, 1951, pp 481-492.

12. P. Wolfe, "The Simplex Method for Quadratic Programming", Econometrica, vol. 27, 3, pp 382-398, July 1959.

13. M. A. Kuliasha, "A Dynamic Model of Power System Operation Incorporating Load Control", Report for Contract No. W-7405-eng 26, prepared by the Oak Ridge National Laboratory.

14. F. A. Viramonteo and H. B. Hamilton, "Optimal Long Range Hydro Scheduling in the Integrated Power System", IEEE Transactions on Power Apparatus and Systems, vol. PAS-97, no. 1, Jan/Feb 1978, pp 292-299.

15. S. Vemuri and E. F. Hill, "Sensitivity Analysis of Optimum Operation of Hydro-Thermal Plants", IEEE Transactions on Power Apparatus and Systems, vol. PAS-96, no. 2, March/April 1977, pp 688-696.

16. R. A. Fernandes and H. D. Philipp, "Hydrogen Cycle Peak-Shaving on the New York State Grid Using Fuel Cells", IEEE Transactions on Power Apparatus and Systems, vol. PAS-96, no. 2, March/April 1977, pp 467-477.

17. C. DeChamps, R. Nuytten, S. T. Lee, "Optimal Operation of Storage Devices in a Power System", paper A80 085-1 presented at the IEEE-PES Winter Meeting, New York, NY, February 3-8, 1980.

18. F. Delebecque and J. P. Quadrat, "Contribution of Stochastic Control Singular Perturbation Averaging and Team Theories to an Example of Large-Scale Systems: Management of Hydropower Production", IEEE Transactions on Automatic Control, vol. AC-23, no. 2, April, 1978, pp 209-222.

19. G. Cohen, "Optimization by Decomposition and Coordination: A Unified Approach", IEEE Transactions on Automatic Control, vol. AC-23, no. 2, April, 1978, pp 222-232.

20. H. H. Happ, R. C. Johnson, and W. J. Wright, "Large Scale Hydro-Thermal Unit Commitment-Method and Results", IEEE Transactions on Power Apparatus and Systems, vol. PAS- , no. , pp 1373-1384.

21. V. H. Quintana and A. Y. Chikhani, "A Stochastic Model for Mid-Term Operation Planning of Hydro-Thermal Systems with Random Reservoir Inflows", paper F80 240-2 presented at the IEEE-PES Winter Meeting, New York, NY February 3-8, 1980.

22. R. Anelli, A. Garzillo, M. Innorta, "Optimization of a Very Large Hydroelectric System by Network Flow Techniques with Additional Constraints", Proceedings of the 1979 IEEE PICA Conference, Cleveland, Ohio, May 15-18, 1979.

23. S. T. Lee, F. Lecaros, J. Peschon, "Application of Dynamic Programming Successive Approximation to Energy Storage Scheduling", IEEE Systems, Man and Cybernetics Society Conference, Denver, Colorado, October 8-10, 1979.

24. C. R. Gagnon, J. F. Bolton, "Optimal Hydro Scheduling at the Bonneville Power Administration", IEEE Transactions on Power Apparatus and Systems, vol. PAS-97, no. 3, pp 772-776, May/June 1978.

25. T. N. Saha, S. A. Khaparde, "An Application of a Direct Method to the Optimal Scheduling of Hydrothermal System", IEEE Transactions on Power Apparatus and Systems, vol. PAS-97, no. 3, pp 977-983, May/June 1978.

26. W. Wood, P. Yatcko, M. P. Bhavaraju, "Economic Assessment of the Utilization of Fuel Cells in Electric Utility Systems", IEEE Transactions on Power Apparatus and Systems, vol. PAS-97, no. 5, pp 1805-1813, Sept/Oct 1978.

27. N. V. Arvanitidis and J. Rosing, "Composite Representation of a Multireservoir Hydroelectric Power System", IEEE Transactions on Power Apparatus and Systems, vol. PAS-89, no. 2, pp 319-326, February 1970.

28. N. V. Arvanitidi and J. Rosing, "Optimal Operation of Multireservoir Systems Using a Composite Representation", IEEE Transactions on Power Apparatus and Systems, vol. PAS-89, no. 2, pp 327-335, February 1970.

29. J. D. C. Little, "The Use of Storage Water in a Hydroelectric System", Journal of the Operations Research Society of America, Vol III, pp 187-197, 1955.

30. F. A. Viramontes, H. B. Hamilton, "Scheduling of Hydro Electric Storage Plants Using Dynamic Programming", Paper A77 509-3 presented at the 1977 IEEE-PES Summer Meeting, Mexico City, Mexico, July 17-22, 1977.

31. S. Soares, C. Lyra, H. Tavares, "Optimal Generation Scheduling of Hydrothermal Power Systems", IEEE Transactions on Power Apparatus and Systems, vol. PAS-99, no. 3, pp 1107-1118, May/June 1980.

32. K. C. Drinnon, L. H. Luallen, and C. C. Whittaker, "Remote Control of Tennessee Valley Authority's Raccoon Mountain Pump Storage Plant", IEEE Transactions on Power Apparatus and Systems, vol. PAS-94, no. 3, pp 834-837, May/June 1975.

33. A. Di Perma, E. Ferrara, "Optimum Scheduling of a Hydroelectric System by Linear Programming", Proceedings of the 1966 Power Systems Computation Conference, Stockholm, Sweden, 1966.

34. H. W. Kuhn and A. W. Tucker, "Nonlinear Programming", Second Berkeley Symposium on Mathematical Statistics and Probability, Berkeley, California, University of California Press, pp 481-492, 1950.

35. F. J. Rees, R. E. Larson, "Computer Aided Dispatching and Operations Planning for an Electric Utility with Multiple Types of Generation", IEEE Transactions on Power Apparatus and Systems, vol. PAS-90, no. 2, pp 891-899, March/April 1971.

35. A. P. Meliopoulos, "A Decomposition Technique to Determine Optimal Coordinating Policies of Energy Storage Plants with Electric Power Systems", Proceedings of the 185h IEEE Conference on Decision and Control, pp 876-878, Fort Lauderdale, Florida, December 12-14, 1979.

37. A. P. Bonaert, A. H. El-Abiad, A. J. Koivo, "Optimal Scheduling of Hydro-Thermal Power Systems", IEEE Transactions on Power Apparatus and Systems, vol. PAS-91, no. 1, pp 263-270, Jan/Feb 1972.

38. A. P. Meliopoulos, "Optimal Coordination of Energy Storage Facilities with an Integrated Electric Power System", Progress Report, NSF Grant No. ENG-7816496.

39. C. W. Watchorn, "Coordination of Hydro and Steam Generation", AIEE Transactions vol. 74, pt. III, pp 142-150, April 1955.

40. R. E. Larson, A. J. Korsak, "A Dynamic Programming Successive Approximations Technique with Convergence Proofs", Automatica, vol. 6, pp 245-252, Pergamon Press, 1970.

41. C. D. Galloway, L. K. Kirchmayer, W. D. Marsh, and A. G. Mellor, "An Approch to Peak Load Economics", AIEE Transactions on Power Apparatus and Systems, vol. 79, pt. III, pp 527-535, August, 1960.

42. R. J. Ringlee, "Bounds for Convex Variational Programming Problems Arising in Power System Scheduling and Control", IEEE Transactions on Automatic Control", vol. AC-10, pp 28-35, January, 1965.

43. G. H. McDaniel, A. F. Gabrielle, "Dispatching Pumped Storage Hydro", IEEE Transactions on Power Apparatus and Systems, vol. PAS-85, pp 465-471, May, 1966.

44. R. J. Cypser, "Computer Search for Economical Operation of a Hydrothermal Electric System", AIEE Transactions on Power Apparatus and Systems, vol. 73, pt. III, pp 1260-1267, October, 1954.

45. C. W. Watchorn, "Some General Basic Planning Characteristics of Pumped Storage Hydro Capacity", IEEE Transactions on Power Apparatus and Systems, pp 464-481, June, 1965.

46. P. J. Bernard, J. F. Dopazo, C. W. Stagg, "A Method for Economic Scheduling of a Combined Pumped Hydro and Steam Generating System", IEEE Transactions on Power Apparatus and Systems, pp 23-30, January, 1964.

47. R. H. Hicks, C. R. Gagnon, S. L. S. Jacoby, and J. S. Kowalik, "Large Scale Nonlinear Optimization of Energy Capability for the Pacific Northwest Hydroelectric System", IEEE Transactions on Power Apparatus and Systems, vol. PAS-93, no. 5, pp 1604-1612, Sept/Oct 1974.

48. D. Teneketzis, B. Sridhar, "Spatial Decomposition and Stochastic Optimization in Multi-Reservoir Systems", Proceedings of the 19th IEEE Conference on Decision and Control, pp 163-165, Albuquerque, NM, December 10-12, 1980.

49. D. Teneketzis, H. Javid, B. Sridhar, "Control of Weakly-Coupled Markov Chains", Proceedings of the 196h IEEE Conference on Decision and Control, pp 137-142, Albuquerque, NM, December 10-12, 1980.

50. IEEE Committee Report, "Bibliography on Pumped Storage to 1975", IEEE Transactions on Power Apparatus and Systems, vol PAS-95, no. 3, pp 839-850, May/June 1976.

BIBLIOGRAPHY

Load Forecasting

1. IEEE Committee Report, "Load Forecast Bibliography Phase I", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-99, No. 1, pp 53-58, Jan/Feb 1980.

2. K. A. Clements, A. J. Wood, "Error Analysis of Econometric Load Forecasting Models", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-98, No. 2, pp 393-399, March/April 1979.

3. G. Singh, K. K. Biswas, A. K. Mahalanabis, "Load Modelling for Real Time Monitoring of Power Systems", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-96, No. 6, pp 1908-1914, Nov/Dec 1977.

4. M. S. Sachdev, R. Billinton, C. A. Peterson, "Representative Bibliography on Load Forecasting", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-96, No. 2, pp 697-700, March/April 1977.

5. M. B. Priestley, T. S. Rao, and H. Tong, "Applications of Principal Component Analysis and Factor Analysis in the Identification of Multivariable Systems", IEEE Transactions on Automatic Control, Vol. AC-19, No. 6, pp 730-734, December 1974.

6. H. Akaike, "Maximum Likelihood Identification of Gaussian Autoregressive Moving Average Models", Biometrica, Vol. 60, No. 2, pp 255-265, 1973.

7. S. Vemuri, E. F. Hill, R. Balasubramanian, "Load Forecasting Using Stochastic Models", paper presented at the 1976 IEEE Winter Meeting, New York, NY, February 1976.

8. W. R. Christiaanse, "Short Term Load Forecasting Using General Exponential Smoothing", Trans. IEEE, Vol. PAS-90, No. 2, pp 900-910, March/April 1971.

9. D. P. Lijesen, and J. Rosing, "Adaptive Forecasting of Hourly Loads Based in Load Measurements and Weather Information", Trans. IEEE, Vol. PAS-90, No. 4, pp 1757-1776, July/August 1971.

10. K. N. Stanton, "Forecasting by Probability Methods", Trans. IEEE, Vol. PAS-90, pp 1183-1189, May/June 1971.

11. S. Vemuri et al., "Load Forecasting Using Stochastic Models", Paper No. TP-1-A Proceedings of the 8th PICA Conference, Minneapolis, Minnesota, pp 31-37.

12. W. R. Christiaanse, "Short Term Load Forecasting Using General Exponential Smoothing", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-90, No. 2, March/April 1971.

13. A. Keyhani, and Rad, T. Eliassi, "A Simulation Study for Recursive Prediction of Hourly Load Demands", Proc. of 1977 PICA Conference, Toronto, Ontario, Canada, May 1977, pp 228-236.

14. K. Srinivasan, and R. Pronovost, "Short Term Load Forecasting Using Multiple Correlation Models", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-94, No. 5, pp 1854-1858, Sept/Oct 1975.

15. A. Ferrandiz, M. Penel, and Y. Pioger, "Short, Medium and Long Term Load Forecasting Models", Paper No. A-75-430-9, Presented at the Summer Meeting of the IEEE Power Engineering Society, San Francisco, CA July 1975.

16. K. L. S. Sharma, and A. K. Mahalanabis, "An Application of Non-Linear Adaptive Estimation Theory in Short-Term Load Prediction", Paper No. C-74-33-1, Presented at the Summer Meeting of the IEEE Power Engineering Society, Anaheim, CA, July 1974.

17. S. L. Corpening, N. D. Reppen, and R. J. Ringlee, "Experience with Weather Sensitive Load Models for Short and Long Term Forecasting", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-92, No. 6, pp 1966-1972, Nov/Dec 1973.

18. K. L. S. Sharma, and A. K. Mahalanabis, "Application of Adaptive Estimation Technique for Short-Term Load Prediction", Paper No. C-72-455-4, IEEE Power Engineering Society Summer Meeting, San Francisco, CA, July 1972.

19. M. S. Sachdev, and S. A. Ibrahim, "Short-Term On-Line Load Forecasting", Paper No. C-72-454-7, IEEE Power Engineering Society Summer Meeting, San Francisco, CA, July 1972.

20. F. D. Galiana, and F. C. Schweppe, "A Weather Dependent Probabilistic Model for Short Term Load Forecasting and Anomaly Detection", Paper No. C-72-171-2, IEEE Winter Power Meeting, New York, January 30-February 4, 1972.

21. P. C. Gupta, and K. Yamada, "Adaptive Short-Term Forecasting of Hourly Loads Using Weather Information", Trans. IEEE, PAS-91, pp 2085-2094, Sept/Oct 1972.

22. C. E. Asbury, "Weather Load Model for Electric Demand and Energy Forecasting", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-94, No. 4, pp 1111-1116, July/August 1975.

23. R. P. Thompson, "Weather Sensitive Electric Demand and Energy Analysis on a Large Geographically Diverse Power System - Application to Short Term Hourly Electric Demand Forecasting", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-95, No. 1, pp 385-393, Jan/Feb 1976.

24. M. Davies, "Techniques for Load Prediction in the Electrically-Supply Industry and Developments of On-Line Load Prediction Techniques with Results from Trials in the Southwest Region of the CEGB", Proc. IEEE, Vol. 117, No. 4, April 1970.

25. E. D. Farmer, M. J. Potton, "Development of On-Line Load Prediction Techniques with Results from Trials in the Southwest Region of the CEGB", Proc. IEEE, Vol. 115, pp 1549-1558, October 1968.

26. K. L. Äström, T. Söderström, "Uniqueness of the Maximum Likelihood Estimate of the Parameters of an ARMA Model", IEEE Transactions on Automatic Control, Vol. AC-19, No. 6, pp 769-773, December 1974.

27. I. Kaufman, "The Inversion of the Vander Monde Matrix and the Transformation to the Jordan Canonical Form", IEEE Transactions on Automatic Control, Vol. AC-14, pp 774-777, December 1976.

28. S. Makridakis, S. C. Wheelwright, Forecasting: Methods and Applications, John Wiley, 1978.

29. G. E. P. Box, G. M. Jenkins, Time Series Analysis: Forecasting and Control, Revised Edition, Holden Day, 1976.

BIBLIOGRAPHY

Technical Characteristics/Operation

1. R. H. Waltman, R. E. Howell, "Jocassee Pumped Storage Project - Startup Testing and Operating Experience", IEEE Transactions on Power Apparatus and Systems, vol. PAS-94, no. 4, pp 1410-1416, July/August 1975.

2. W. T. Alley, "Hydroelectric Plant Capability Curves", IEEE Transactions on Power Apparatus and Systems, vol. PAS-96, no. 3, pp 999-1003, May/June 1977.

3. A. T. L. Murray, H. Rogne, "Foyers Pumped Storage Scheme", IEEE Transactions on Power Apparatus and Systems, vol. PAS-98, no. 5, pp 1739-1749, Sept/Oct 1979.

4. G. P. Baer, H. Jacobs, "Automation of Wehr Pumped - Storage Plant Using Process Computers", IEEE Transactions on Power Apparatus and Systems, vol. PAS-

5. P. G. Brown, "Reduced Voltage Starting Performance of Synchronous Condensers and PUmped Hydro Units", IEEE Transactions on Power Apparatus and Systems, vol. PAS-95, no. 2, pp 505-511, March/April 1976.

6. R. H. S. Hardy, M. T. Sulatisky, W. B. H. Cooke, "Residential Heat Storage Furnaces for Load Management - Design and Control", IEEE Transactions on Power Apparatus and Systems, vol. PAS-99, no. 2, pp 417-425, March/April 1980.

7. J. W. Beck, D. P. Carroll, G. E. Gareis, P. C. Krause, and C. M. Ong, "A Computer Study of Battery Energy Storage and Power Conversion Equipment Operation", IEEE Transactions on Power Apparatus and Systems, vol. PAS-95, no. 4, pp 1064-1072, July/August 1976.

8. P. Biro, "Electrical Features of Fairfield Pumped Storage Project, South Carolina Electric and Gas Company", IEEE Transactions on Power Apparatus and Systems, vol. PAS-99, no. 5, pp 1752-1757, Sept/Oct 1980.

9. F. P. Bevc, R. J. Meehan, "Generator-Motors for Pacific Gas and Electric's Helms Pumped Storage Project", paper F80211-3 presented at the IEEE-PES Winter Meeting, New York, NY, February 3-8, 1980.

10. C. Harvey, III, "Concepts and Features of the Automated Unit Control System for the Raccoon Mountain Pumped Storage Plant", IEEE Transactions on Power Apparatus and Systems, vol. PAS-97, no. 2, pp 513-519, March/April 1978.

11. R. V. Snow, "Design of the Battery Energy Storage Test Facility", IEEE Transactions on Power Apparatus and Systems, vol. PAS-98, no. 1, pp 300-303, Jan/Feb 1979.

12. G. A. Phillips, J. W. Walton, F. J. Kornbrust, "Progress in Self-Commutated Inverters for Fuel Cells and Batteries", IEEE Transactions on Power Apparatus and Systems, vol. PAS-98, no. 4, pp 1466-1475, July/August 1979.

13. IEEE Committee Report, "Protective Relaying for Pumped Storage Hydro Units Rotating Machinery Protection Subcommittee of the IEEE Power System Relaying Committee", IEEE Transactions on Power Apparatus and Systems, vol. PAS-94, no. 3, pp 899-907, May/June 1975.

14. H. A. Peterson, N. Mohan, R. W. Boom, "Superconductive Energy Storage Inductor-Converter Units for Power Systems", IEEE Transactions on Power Apparatus and Systems, vol. PAS-94, no. 4, pp 1337-1348, July/August 1975.

15. M. H. Fisher, G. W. Fonseca, S. M. Gilbert, "Design Features of Mt. Elbert Pumped-Storage Powerplant", IEEE Transactions on Power Apparatus and Systems, vol. PAS-94, no. 6, pp 2170-2176, Nov/Dec 1975.

16. M. I. Olken, N. Petroro, "Smith Mountain Unit 3 Control and Protection of a Pumped Storage Unit", IEEE Transactions on Power Apparatus and Systems, vol. PAS-99, no. 6, pp 2369-2372, Nov/Dec 1980.

17. F. P. Bevc and R. J. Meehan, "Generator-Motors for Pacific Gas and Electric's Helms Pumped Storage Project", IEEE Transactions on Power Apparatus and Systems, vol. PAS-99, no. 6, pp 2021-2030, Nov/Dec 1980.

BIBLIOGRAPHY

Proposed Energy Storage Schemes

1. V. T. Sulzberger, J. Zenkoski, "The Potential for Application of Energy Storage Capacity on Electric Utility Systems in the United States, Part I", IEEE Transactions on Power Apparatus and Systems, vol. PAS-95, no. 6, pp 1872-1881, Nov/Dec 1976.

2. V. T. Sulzberger, J. Zemkoski, "The Potential for Application of Energy Storage Capacity on Electric Utility Systems in the United States, Part II", IEEE Transactions on Power Apparatus and Systems, vol. PAS-96, no. 1, pp 213-221, Jan/Feb 1977.

3. J. W. Leslie, "Hydroelectric Power in New England", IEEE Transactions on Power Apparatus and Systems, vol. PAS-98, no. 4, pp 1354-1357, July/August 1979.

4. R. Ramakumar, H. J. Allison, W. L. Hughes, "Solar Energy Conversion and Storage Systems for the Future", IEEE Transactions on Power Apparatus and Systems, vol. PAS-94, no. 6, pp 1926-1934, Nov/Dec 1975.

5. C. K. Haswell, et al, "Pumped Storage and Tidal Power in Energy Systems", ASCE Proceedings, vol. 98, PO2, no. 9297, pp 201-220, October, 1972.

6. T. L. Shaw, G. R. Thorpe, "Integration of Pumped-Storage with Tidal Power", ASCE Proceedings, vol. 98, PO1, pp 159-180, January, 1971.

7. G. R. Thorpe, T. L. Shaw, "The Case for Low-Head Estuarial Pumped Storage", Water Power, vol. 22, pp 107-108, March 1970.

8. "Hydro-Electric Power Resources of the United States: Developed and Undeveloped", Document No. 015-000-00347-2, 1976.

9. IEEE Committee Report, "Survey of Pumped Storage Projects in the United States and Canada to 1975", IEEE Transactions on Power Apparatus and Systems, vol. PAS-95, no. 3, pp 851-858, May/June 1976.

10. A. E. Allen, "Potential for Conventional and Underground Pumped-Storage", IEEE Transactions on Power Apparatus and Systems, vol. PAS-96, no. 3, pp 993-998, May/June 1977.

BIBLIOGRAPHY

Power System Simulation Methodologies

1. R. R. Booth, "A Computer Model for the Simulation of Power System Operations", PSCC Proceedings, 1969.

2. R. R. Booth, "Power System Simulation Model Based on Probability Analysis", IEEE Transactions on Power Apparatus and Systems, vol. PAS-91, no. 1, pp Jan/Feb 1972.

3. D. S. Joy and R. T. Jenkins, "A Probabilistic Model for Estimating the Operating Cost of an Electric Power Generating System", presented at the Nuclear Utilities Planning Methods Symposium, Chattanooge, Tennessee, January 16-18, 1974.

4. M. A. Sager, R. J. Rignlee, and A. J. Wood, "A New Generation Production Cost Program to Recognize Forced Outages", IEEE Transactions on Power Apparatus and Systems, vol. PAS-91, no. 5, pp 2114-2124, Sept/Oct 1972.

5. M. A. Sager and A. J. Wood, "Power System Production Cost Calculations - Sample Studies Recognizing Forced Outages", IEEE Transactions on Power Apparatus and Systems, vol. PAS-92, no. 4, pp 154-158, Jan/Feb 1973.

6. A. P. Meliopoulos, "Computer Aided Instruction of Energy Source Utilization Problems", to appear in the IEEE Transactions on Education, special issue on energy problems, 1981.

7. N. S. Rau, P. Toy, and K. F. Schenk, "Expected Energy Production Costs by the Method of Moments", IEEE Transactions on Power Apparatus and Systems, vol. PAS-99, no. 5, pp 1980-1917, Sept/Oct 1980.

8. J. P. Stremel, R. T. Jenkins, R. A. Babb, and W. D. Bayless, "Production Costing Using the Comulant Method of Representing the Equivalent Load Curve", IEEE Transactions on Power Apparatus and Systems, vol. PAS-99, no. 5, pp 1947-1955, Sept/Oct 1980.

9. J. T. Day, "Forecasting Minimum Production Costs with Linear Programming", IEEE Transactions on Power Apparatus and Systems, vol. PAS-90, no. 1, pp 814-823, March/April 1971.

10. A. P. Meliopoulos, G. Cokkinides, N. Skordilis, "Production Costing Methodologies", report to Westinghouse Electric Corporation, Project E21-647, Georgia Institute of Technology, Atlanta, Georgia, August, 1978.

11. A. S. Debs, A. P. Meliopoulos, R. Bermudez, "Probabilistic Production Costing", report to Westinghouse electric Corporation, Project E21-647, Georgia Institute of Technology, Atlanta, Georgia, 1977.

12. A. P. Meliopoulos, G. J. Cokkinides, "Probabilistic Production Costing Methodologies", Final Report to Westinghouse Electric Corporation, Georgia Institute of Technology, Atlanta, Georgia, September, 1979.

13. W. Sager and A. J. Wood, "Electric Utility Corporate Models", Proceedings of 4th PSCC Conference, Grenoble, France, 1972.

14. F. J. Rees and J. M. Goodrich, "Stochastic Production Costing Security Assessment and Maintenance Scheduling of a Large System of Hydro-Thermal-Nuclear Plants", JACC Conference Proceedings, paper no. 2-A4, August, 1971.

BIBLIOGRAPHY

General

1. P. H. Henault, "On the Application of Dynamic Programming to Long-Range Planning with an Uncertain Future", Ph.D. Dissertation, Department of Industrial Engineering at Stanford University, 1969.

2. G. F. Reid and L. Hasdorff, "Economic Dispatch Using Quadratic Programming", IEEE Transactions Power App. Syst., vol. PAS-92, Nov/Dec 1973, pp 2015-2023.

3. A. M. H. Rashed and D. H. Kelly, "Optimal Load Flow Solution Using Lagragian Multipliers and the Hessian Matrix", paper T74 160-8 presented at the IEEE PES Winter Meeting, New York, NY, January 27-February 1, 1974.

4. H. H. Happ, "Optimal Power Dispatch - A Comprehensive Survey", IEEE Transactions on Power Apparatus and Systems, vol. PAS-96, no. 3, May/June 1977.

5. N. Nabona and L. L. Freris, "Optimization of Economic Dispatch through Quadratic and Linear Programming", Proc. Inst. Elec. Eng., vol. 120, May 1973, pp 574-579.

6. H. W. Dommel, W. F. Tinney, "Optimal Power Flow Solutions", IEEE Transactions on Power Apparatus and Systems, vol. PAS-87, pp 1866-1876, October, 1968.

7. A. M. Sasson, F. Biloria, and F. Aboytes, "Optimal Load Flow Solution Using the Hessian Matrix", IEEE Transactions on Power Apparatus and Systems, vol. PAS-92, Jan/Feb 1973, pp 31-41.

8. B. Stott, "Review of Load Flow Calculation Methods", Proc. IEEE, vol. 62, no. 7, pp 916-929, July 1974.

BIBLIOGRAPHY

Books

1. R. Larson, State Increment Dynamic Programming, American Elsevier Publishing Company, New York, NY, 1967.

2. E. J. Hannan, Time Series Analysis, London: Menthen, London, 1960.

3. K. J. Astrom, Introduction to Stochastic Control Theory, New York, Academic Press, 1970.

4. L. K. Kirchmayer, Economic Control of Interconnected Systems, John Wiley and Sons, Inc., 1959.

5. S. E. Dreyfus and A. M. Law, The Art and Theory of Dynamic Programming, Academic Press, 1977.

6. R. Bellman, Dynamic Programming, Princeton, NJ, Princeton University Press, 1957.

7. George L. Hemhauser, Introduction to Dynamic Programming, (New York: John Wiley and Sons, 1966).

8. M. E. El-Hawary, G. S. Christensen, Optimal Economic Operation of Electric Power Systems, Academic Press, NY, 1979.

9. R. Billinton, Power System Reliability Evaluation, Gordon and Breach, Science Publishers, Inc., 1970.

10. H. Karreman (Editor), Stochastic Optimization and Control, John Wiley and Sons, New Yor, NY, 1968.

11. A. Papoulis, Probability, Random Variables and Stochastic Processes, McGraw-Hill, 1965.

12. E. Parzen, Stochastic Processes, Holden Day, Inc., San Francisco, CA, 1962.

13. G. Hadley, Nonlinear and Dynamic Programming, Addison-Wesley Publishing Co., Reading, MA, 1964.

14. G. B. Dantzig, Linear Programming and Extensions, Princeton University Press, Princeton, NJ, 1963.

15. C. W. Stagg, A. H. El-Abiad, Computer Methods in Power System Analysis, McGraw-Hill Book Company, New York, NY, 1968.

VII. PERSONNEL

BIOGRAPHICAL SKETCH

MELIOPOULOS, ATHANASIOS P. -- Assistant Professor of Electrical Engineering
Georgia Institute of Technology

EDUCATION

ME and EE Diploma, National Technical University of Athens, Greece	1972
MSEE, Georgia Institute of Technology	1974
Ph.D., (E.E.) Georgia Institute of Technology	1976

EMPLOYMENT HISTORY

Western Electric	
Exchange Student	Summer 1971
Georgia Institute of Technology	
Teaching and Research Assistant	1972-1976
Assistant Professor of Electrical Engineering	1976-present

EXPERIENCE SUMMARY

At Georgia Tech he is presently involved in teaching courses on operations research, applications of probability theory in power systems and on control, operation, and planning of power systems. At the research level, he presently holds a grant from NSF to study the optimal coordination of Energy Storage Facilities with Electric Power Systems. He is also the co-principal investigator on other research projects titled 1) "Development techniques for Analysis and Design of Power Substation Ground Systems," sponsored by EPRI, 2) "Bulk Power Transmission Reliability Evaluation," sponsored by Southern Services, and 3) "Multiphase Power Flows," sponsored by Westinghouse Electric Corporation.

His current research activities are on: dynamic simulation of power systems, electromagnetic transient analysis, probabilistic methods of production costing of electric power systems, including energy storage facilities applications of control theory to power system problems, long range optimal planning of electric power transmission networks, advanced analysis techniques of electric power systems, analysis of grounding systems and minicomputer applications to large scale problems.

As a Research Assistant at Georgia Tech, he has developed a computer program for optimal protection coordination of distribution systems under contract NAS10-8375. He has also developed and implemented a technique for optimal long range planning of electric power transmission networks (project E21-647, sponsored by Westinghouse Electric Corporation). He has also implemented and optimized large scale load flow programs for minicomputer applications.

SCIENTIFIC AND PROFESSIONAL SOCIETIES

Member: Sigma Xi
Member: Hellenic Society of Professional Engineers
Member: IEEE, PES

CURRENT FIELDS OF INTEREST

Applications of systems theory and operations research to large scale systems with emphasis on power systems; power system dynamic analysis, electromagnetic transient analysis, power systems planning, power system grounding and power system control. Minicomputer and microcomputer applications to large scale systems.

DISSERTATION

A. P. Meliopoulos, "A General Approach to the Planning of a Transmission Network," Ph.D. Dissertation, Georgia Institute of Technology, 1976. (Advisor: Dr. A. S. Debs)

PUBLICATIONS

1. Books

Problems and Concise Theory of High Voltage Structures, (in Greek), B. H. Sellountos Publishing Co., Athens, Greece, 1972.

2. Journal Papers

H. N. Nunnally, R. P. Webb, E. B. Joy and A. P. Meliopoulos, "Computer Simulation for Determining Step and Touch Potentials Resulting from Faults or Open Neutrals in URD Cable," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-98, No. 3, pp. 1130-1136, May/June 1979.

E. B. Joy. A. P. Meliopoulos, and R. P. Webb, "Touch and Step Calculation for Substation Grounding Systems," IEEE Transactions on Power Apparatus and Systems (abstract form), Vol. PAS-98, No. 4, July/August 1979.

A. P. Meliopoulos, R. P. Webb, E. B. Joy, Discussion of paper "Numerical Computation of the Grounding Resistance of Substations and Towers", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-99, No. 3, pp. 963-964, May/June 1980.

A. P. Meliopoulos, R. P. Webb, and E. B. Joy, "Analysis of Grounding Systems," paper presented at the 1980 IEEE PES Winter Meeting and accepted for publication in the IEEE Transactions on Power Apparatus and Systems.

A. P. Meliopoulos, R. P. Webb, and E. B. Joy, "Computer Simulation of Faulted URD Cables: Analysis and Results." paper presented at the 1980 IEEE-PES Summer Meeting and accepted for publication in the IEEE Transactions on Power Apparatus and Systems.

A. P. Meliopoulos, "Computer Aided Instruction of Energy Source Utilization Problems", paper submitted and accepted for publication in the IEEE Transactions on Education.

3. Conference Proceedings

R. P. Webb, A. P. Meliopoulos. and M. Wright, "Protection Coordination for the Kennedy Space Center Electric Distribution Network," Proceedings of the 1974 Protective Relaying Conference, Atlanta, Georgia, May 1974.

A. P. Meliopoulos, A. S. Debs, R. P. Webb, and L. J. Rindt, "A Nonlinear Branch and Bound Algorithm Applied to optimal Long Range Power System Transmission Expansion Planning," Proceedings of the 1976 IEEE Conference on Decision and Control, pp. 255-260, Clearwater, Florida, Dec. 1-3, 1976.

A. P. Meliopoulos and A. S. Debs, "Impact of Improved On-Line Security Dispatching on Long-Term Transmission Expansion Planning," Proceedings of the 1977 Power Industry Computer Application Conference (PICA X), Toronto, Ontario, Canada, May 24-27, 1977.

E. B. Joy, W. J. Barnes, A. P. Meliopoulos. H. N. Nunnally, R. P. Webb, and R. E. Wilson, "Multi-Layer Method of Moments Analysis of Earth Grounding Systems," Proceedings of Southeastcon '78, pp. 12-14, Atlanta, Georgia, April 10-12, 1978.

A. P. Meliopoulos, N. Skordilis. "Coordination of Pumped Storage Stations with the Integrated Electric Power System," Proceedings of the 1979 Control of Power Systems Conference, pp. 127-131, College Station, Texas, March 19-21, 1979.

A. P. Meliopoulos, D. P. Rudolph, R. P. Webb, and E. B. Joy, "URD Cable Analysis to Determine Shield and Earth Potentials in Normal and Abnormal Operating Conditions," Proceedings of the 1979 Southeastcon, pp. 394-8, Roanoke, Virginia, April 2-4, 1979.

A. P. Meliopoulos, "A Decomposition Technique to Determine Optimal Coordinating Policies of Energy Storage Plants with Electric Power Systems," Proceedings of the 18th IEEE Conference on Decision and Control, pp. 876-878, Fort Lauderdale, Florida, December 12-14, 1979.

R. J. Bennon, J. A. Juves, and A. P. Meliopoulos, "Use of Sensitivity Analysis in Automated Transmission Planning," paper to be presented at the 1981 Power Industry Computer Applications Conference, Philadelphia, PA, May 5-8, 1981.

MAJOR RESEARCH REPORTS

A. P. Meliopoulos and R. P. Webb, "Protection Coordination of the Kennedy Space Center Electric Distribution Network," report to NASA, Project NA 10-8375, Georgia Institute of Technology, Atlanta, Georgia, June 1974.

R. P. Webb, A. S. Debs, C. Alford, L. Cibulka. T. Hayes, A. P. Meliopoulos, and L. Wakefield. "Development of Computer Based Power System Analysis Techniques," report to Westinghouse Electric Corporation, Project E-21-647. Georgia Institute of Technology, Atlanta, Georgia. June 1975.

A. P. Meliopoulos and A. S. Debs, "Automatic Transmission Planning," report to Westinghouse Electric Corporation, Project E-21-647, Georgia Institute of Technology, Atlanta, Georgia, June 1976.

A. P Meliopoulos and R. P. Webb, "Fast Decoupled Load Flow Optimization for Minicomputer Application," report to Westinghouse Electric Corporation, Project E-21-647, Georgia Institute of Technology, Atlanta, Georgia, July 1977.

A. S. Debs, A. P. Meliopoulos, and R. Bermudez, "Probabilistic Production Costing," report to Westinghouse Electric Corporation, Project E-21-647, Georgia Institute of Technology, Atlanta, Georgia, July 1977.

A. P. Meliopoulos and D. Rudolph, "A Piecewise Fast Decoupled Load Flow Algorithm for Fast Contingency Analysis," report to Westinghouse Electric Corporation, Project E-21-647, Georgia Institute of Technology, Atlanta, Georgia, July 1977.

R. P. Webb, E. B. Joy, H. N. Nunnally. and A. P. Meliopoulos, "Computer Program for Determination of Earth Potentials Due to Faults or Loss of Concentric Neutral on URD Cable," report to EPRI, project 797-1, Georgia Institute of Technology, Atlanta, Georgia, May 1977.

A. P. Meliopoulos, G. Cokkinides, and N. Skordilis, "Production Costing Methodologies," report to Westinghouse Electric Corporation, Project E21-647, Georgia Institute of Technology, Atlanta, Georgia, August 1978.

A. P. Meliopoulos and T. D. Vismor, "Piecewise Fast Decoupled Load Flow for Fast Contingency Analysis and Optimal Load Allocation," report to Westinghouse Electric Corporation, Project E21-647, Georgia Institute of Technology, Atlanta, Georgia, August 1978.

A. P. Meliopoulos and G. J. Cokkinides, "Probabilistic Production Costing Methodologies," Final Report to Westinghouse Electric Corporation, Georgia Institute of Technology, Atlanta, Georgia, September 1979.

A. P. Meliopoulos and B. Subba, "Subtransmission Modeling: Diakoptical Load Flows and Optimal Load Allocation," Final Report to

Westinghouse Electric Corporation, Georgia Institute of Technology, Atlanta, Georgia, September 1979.

E. B. Joy, A. P. Meliopoulos, and R. P. Webb, "Graphical and Tabular Results of Computer Simulation of Faulted URD Cables: Volume I and II," Final Report to EPRI, Georgia Institute of Technology, Atlanta, Georgia, July 1979.

R. P. Webb, A. P. Meliopoulos, and A. Plaz, "Electric Power Bulk Transmission System Reliability Evaluation," Final Report to Georgia Power Company, Georgia Institute of Technology, Atlanta, Georgia, April 1980, with R. P. Webb and A. Plaz.

A. P. Meliopoulos, "Analysis Techniques for Electric Power Grounding Systems," Final Report to Westinghouse Electric Corporation, Georgia Institute of Technology, Atlanta, Georgia, September 1980.

A. P. Meliopoulos, R. P. Webb, and P. Hayet, "Protection Coordination of a Radial Distribution Network." Final Report to Westinghouse Electric Corporation. Georgia Institute of Technology. Atlanta, Georgia, November 1980.

SHORT COURSE LECTURES

1. "Advanced Load Flow Techniques." Southeastern Electric Exchange. Auburn University, Auburn, Alabama, June 10-22, 1979.
2. "Substation Grounding: Practice and Analysis," Georgia Tech Continuing Education Short Course, Atlanta, Georgia, March 18-21, 1980.
3. "Advanced Load Flow Techniques," Southeastern Electric Exchange, Auburn University, Auburn, Alabama, September 8-19, 1980.

CONSULTING ACTIVITIES

Westinghouse Electric Corporation (East Pittsburgh, Pennsylvania)	1976-1978
Southwire, Carrollton, Georgia	1978-1980

RESEARCH CONTRACT/GRANTS - PRINCIPAL/CO-PRINCIPAL INVESTIGATOR

1. "Development of Computer Based Power System Analysis Techniques." Principal Investigators: Roger P. Webb, A. S. Debs, A. P. Meliopoulos
   Sponsor: Westinghouse Electric Corporation
   Amount: $31,648; Cost Sharing (GIT) $10,584
   Duration: September 1976 - August 1977

2. "Development of Computer Based Power System Analysis Techniques"
Principal Investigators: A. P. Meliopoulos, A. S. Debs, R. P. Webb
Sponsor: Westinghouse Electric Corporation
Amount: $31,549; Cost Sharing (GIT) $10,683
Duration: September 1977 - August 1978

3. "Development of Computer Based Power System Analysis Techniques"
Principal Investigator: A. P. Meliopoulos, R. P. Webb
Sponsor: Westinghouse Electric Corporation
Amount: $19,498
Duration: December 1977 - November 1978

4. "Management of Electrical Backup Demand in Solar Heating and Cooling Application"
Principal Investigator: Atif Debs
Faculty Associates: A. P. Meliopoulos, J. R. Williams
Sponsor: ERDA
Amount: $78,161
Involvement: September 1977 - March 1978

5. "Development of Computer Based Power System Analysis Techniques"
Principal Investigators: A. P. Meliopoulos, R. P. Webb
Sponsor: Westinghouse Electric Corporation
Amount: $35,814; Cost Sharing (GIT) $5,808
Duration: September 1978 - September 1979

6. "Optimal Coordination of Energy Storage Facilities with an Integrated Electric Power System"
Principal Investigator: A. P. Meliopoulos
Sponsor: NSF
Amount: NSF $46,330; Cost Sharing (GIT) $4,217
Duration: December 1978 - Present

7. "Graphical and Tabular Results of Computer Simulation of Faulted URD Cables"
Principal Investigators: E. B. Joy, A. P. Meliopoulos, R. P. Webb
Sponsor: Electric Power Research Institute
Amount: $61,170
Duration: June 1978 - July 1979

8. "Bulk Power Transmission Reliability Evaluation"
Principal Investigators: A. P. Meliopoulos, R. P. Webb
Sponsor: Georgia Power Company
Amount: $7,591 (Georgia Power Company), $3,544 (Georgia Tech Cost Sharing)
Duration: August 1979 - February 1980

9. "Development of Computer Based Power System Analysis Techniques,"
Principal Investigators: R. P. Webb, A. P. Meliopoulos
Sponsor: Westinghouse Electric Corporation
Amount: $36,000
Duration: September 1979 - September 1980

10. "Underground Cable Test Simulation and Analysis"
Principal Investigators: R. P. Webb, A. P. Meliopoulos, E. B. Joy
Sponsor: Florida Power and Light Company, Miami, Florida
Amount: $24,927
Duration: 16 months (January 1, 1979 - April 30, 1980)

11. "Development for Techniques for Analysis and Design of Power Substation Ground Systems"
Electric Power Research Institute (EPRI), Palo Alto, California
Principal Investigators: E. B. Joy, A. P. Meliopoulos, R. P. Webb
Amount: $165,413
Duration: 20 months (August 1, 1980 - March 31, 1982)

12. "Multiphase Power Flows"
Sponsor: Westinghouse Electric Corporation, East Philadelphia, Pennsylvania
Principal INvestigators: A.`P. Meliopoulos, R. P. Webb
Amount: $28,747 for one year
Duration: September 1980 - September 1981

13. "Bulk Power Transmission Reliability Evaluation"
Sponsor: Southern Services
Principal Investigators: A. P. Meliopoulos, R. P. Webb
Amount: $89,110 for two years
Duration: January 1981 - January 1983

14. "Development of Interactive Instructional Software for Electromagnetic Transient Analysis"
Sponsor: Georgia Institute of Technology
Principal Investigator: A. P. Meliopoulos
Amount: $1,795 for one academic quarter
Duration: March 1981 - June 1981

February 1981